Effective Stiffness of Reinforced Concrete Columns

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Overview

The stiffnesses of the structural members of a building strongly influence the response of the building to ground shaking. For linear analysis, the member stiffnesses control predictions of the period of the structure, the distribution of loads within the structure, and the deformation demands. For nonlinear analysis, an accurate estimate of the member stiffness is required to reliably estimate the yield displacement, which in turn, affects the predicted displacement ductility demands. In practice, simple procedures are needed to estimate an effective stiffness up to yielding of each structural component.

This research digest uses data from the PEER Structural Performance Database (Berry et al. 2004) to assess the effective stiffness of reinforced concrete columns and compares the results with effective stiffness values provided in the Federal Emergency Management Agency (FEMA) 356 seismic rehabilitation guidelines (ASCE 2000). It is demonstrated that the FEMA 356 procedure substantially overestimates the stiffness of columns with low axial loads, because the effective stiffness of such columns is significantly influenced by deformations resulting from bar slip in the beam-column joints or footings. The digest concludes with practical recommendations for improving estimates of effective stiffness.

Effective Stiffness Model

The yield displacement of a column can be considered as the sum of the displacements due to flexure, bar slip, and shear:

$$\Delta_y = \Delta_{flex} + \Delta_{slip} + \Delta_{shear} \qquad [1]$$

Assuming the column is fixed against rotation at both ends and assuming a linear variation in curvature over the height of the column, the contribution of flexural deformations to the displacement at yield can be estimated as follows:

$$\Delta_{flex} = \frac{L^2}{6}\phi_y = \frac{L^2}{6}\frac{M_{0.004}}{EI_{flex}} \qquad [2]$$

where L is the length of the column, ϕ_y is the yield curvature, and $M_{0.004}$ is the flexural moment at a maximum concrete compressive strain of 0.004, as defined in Figure 1. The effective flexural stiffness of the column, EI_{flex}, can be determined from the moment-curvature relationship shown in Figure 1.

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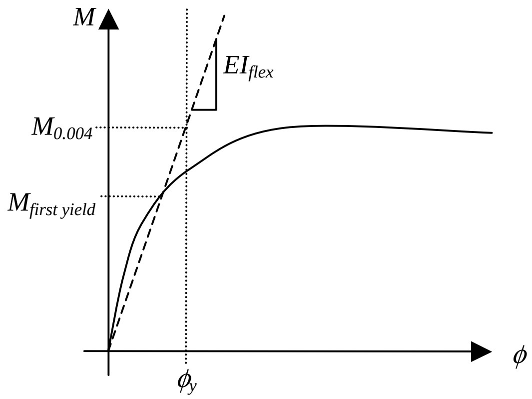

Figure 1: Moment-curvature relationship and definition of yield curvature

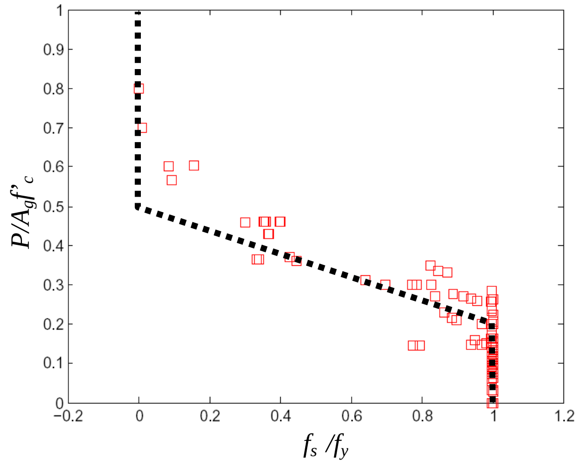

Figure 2: Relationship between axial load and stress in the tension reinforcement

The displacement due to bar slip at yield can be estimated as follows (Elwood and Moehle, 2003):

$$\Delta_{slip} = \frac{L d_b f_s \phi_y}{8u} \qquad [3]$$

where d_b is the diameter of the longitudinal reinforcement, f_s is the stress in the tension reinforcement, and u is the average bond stress between the longitudinal reinforcement and the footing or joint concrete. A bond stress of $u = 6\sqrt{f_c'}$ (psi units) will be assumed in the following calculations (Sozen et al., 1992).

For the purpose of this paper, the "first yield" of a column will be defined as the first point at which the first reinforcing bar yields in tension or the concrete reaches a maximum compressive strain of 0.002. For this condition, the stress in the tension reinforcement, f_s, used in Eq. 3 will vary depending on the axial load on the column. For columns with low axial loads, the tension reinforcement will yield, and hence, f_s can be taken as equal to the yield stress, f_y. The stress in the tension reinforcement will decrease as the axial load on the column increases, reaching zero when the depth of the neutral axis is equal to the effective depth of the column. The variation of f_s with axial load was investigated by considering 120 columns from the PEER Structural Performance Database (Berry et al. 2004). In particular, rectangular columns with normal strength concrete (f'_c < 60 MPa) and standard longitudinal reinforcement arrangements were considered. Figure 2 indicates that while the stress in the tension reinforcement depends on the particular details of each column, the stress can be estimated as equal to the yield stress for axial loads below $P/A_g f'_c = 0.2$ and equal to zero for axial loads above $P/A_g f'_c = 0.5$, with a linear interpolation between these points.

The shear deformation can be estimated as:

$$\Delta_{shear} = \frac{2M_{0.004}}{(AG)_{eff}} \qquad [4]$$

The deformations due to shear prior to yielding are usually very small, except for stocky columns with high levels of shear cracking.

For engineering practice, the response of a column prior to yielding can be approximated as linear-elastic with a single effective stiffness, EI_{eff}:

$$EI_{eff\ calc} = \frac{M_{0.004} L^2}{6\Delta_y} \quad [5]$$

where Δ_y is given by equations 1, 2, 3, and 4. The following section will compare the effective stiffness from Eq. 5, and the flexural stiffness (Figure 1), with the effective stiffness determined from the 120-column dataset.

Comparison of Calculated and Measured Stiffnesses

For each column, the envelope of the measured lateral load-displacement relationship was corrected for P-delta effects to give the effective lateral force on the column. The yield displacement of the column was then determined as shown in Figure 3.

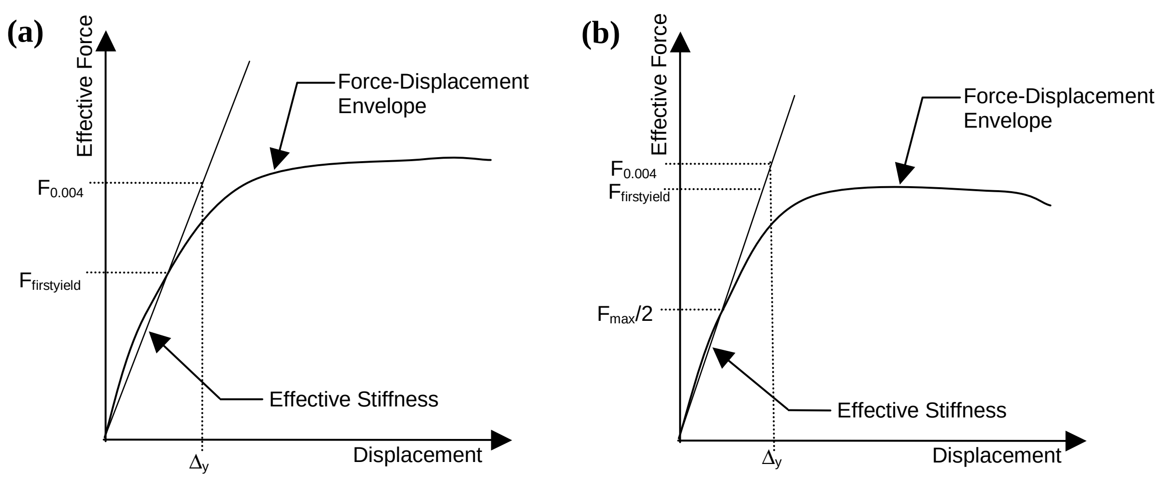

Figure 3: Definition of yield displacement and effective stiffness from test data

For columns where the maximum measured effective force, F_{max}, was at least 105% of the calculated force at first yield (defined by yielding of the tension reinforcement or when the maximum concrete strain reaches 0.002, whichever occurs first), the effective stiffness is defined based on the point on the measured force-displacement envelope corresponding to the calculated force at first yield (Figure 3a). This definition is consistent with the bilinear moment curvature relationship shown in Figure 1. For columns where the maximum measured effective force, F_{max}, was not at least 5% larger than the calculated force at first yield, the effective stiffness was defined based on the point on the measured force-displacement envelope with an effective force equal to F_{max} /2 (Figure 3b). Using the definition of the yield displacement from Figure 3, the measured effective stiffness can be defined as:

$$EI_{eff\ meas} = \frac{F_{0.004}L^3}{12\Delta_y} \quad [6]$$

Figure 4 compares the measured and calculated effective stiffnesses for the 120-column dataset as a function of the normalized axial load. The effective stiffness calculated based on Eqs. 1-5 provides a good estimate of the measured effective stiffness, particularly for columns with axial loads less than $0.5A_g f'_c$.

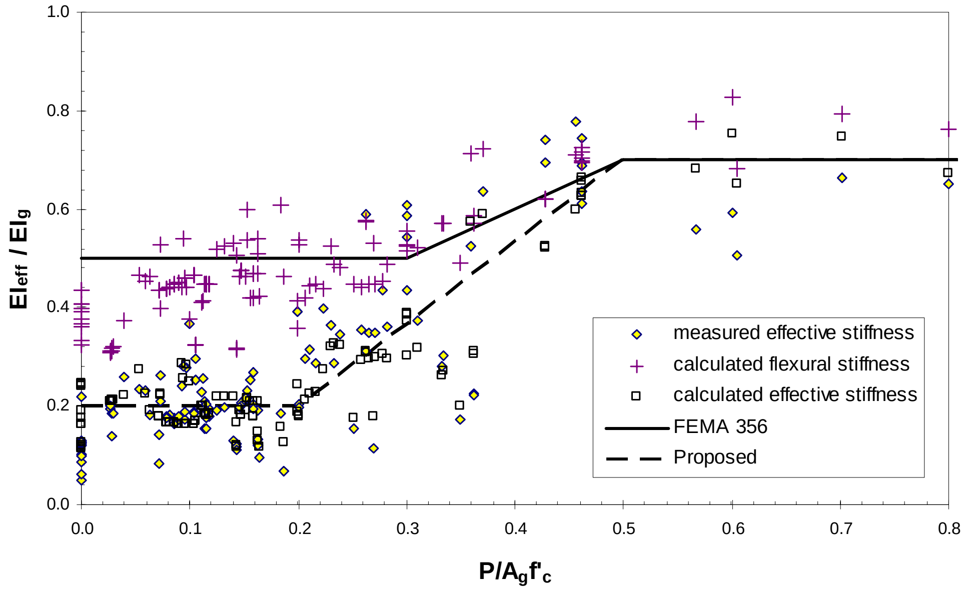

Figure 4: Comparison of calculated and measured effective stiffnesses

The recommended effective stiffness values from FEMA 356 are also shown on Figure 4, and appear to provide a good estimate of the calculated flexural stiffnesses for all the columns, but the FEMA recommendations significantly overestimate the measured effective stiffnesses for columns with axial loads less than $0.3A_g f'_c$. Figure 4 further suggests that slip deformations can account for approximately half of the total deformation at yield for columns with low axial loads, and hence, cannot be ignored when determining the effective stiffness of the column.

Based on the above observations, the following effective stiffness values are proposed for modeling rectangular reinforced concrete columns with normal-strength concrete:

$$EI_{eff} / EI_g = 0.2 \qquad \frac{P}{A_g f_c'} \le 0.2$$

$$= \frac{5}{3}\frac{P}{A_g f_c'} - \frac{4}{30} \qquad 0.2 < \frac{P}{A_g f_c'} \le 0.5 \quad [7]$$

$$= 0.7 \qquad 0.5 < \frac{P}{A_g f_c'}$$

As shown in Figure 4, Eq. 7 is consistent with the FEMA 356 recommendations for axial loads above $0.5A_g f'_c$, but suggests substantially lower stiffnesses for columns with lower axial loads.

Table 1 provides statistics for the ratio of the measured effective stiffness (Eq. 6) to the calculated effective stiffness based on the approaches discussed in this digest. The FEMA 356 recommendations overestimate the measured effective stiffnesses by nearly 100% and result in a coefficient of variation that exceeds 50%. Although substantial scatter in the results still remains, on average the effective stiffness based on Eq. 7 provides a much better estimate of the effective stiffnesses observed for the 120 columns considered in this study.

Table 1: Statistics for the ratio of measured to calculated effective stiffness

Calculated Stiffness Model	**E[** $EI_{eff\ meas} / EI_{eff\ calc}$ **]**	**cov[** $EI_{eff\ meas} / EI_{eff\ calc}$ **]**
Flexural Stiffness (Eq. 2)	0.56	0.46
Total Stiffness (Eq. 5)	1.05	0.28
FEMA 356	0.52	0.53
Proposed model (Eq. 7)	0.99	0.35

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Keywords

Reinforced concrete, columns, stiffness, bond slip, flexural stiffness, FEMA 356