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Inelastic displacement ratios for evaluation of existing structures

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SUMMARY

Results of a detailed statistical study of constant relative strength inelastic displacement ratios to estimate maximum lateral inelastic displacement demands on existing structures from maximum lateral elastic displacement demands are presented. These ratios were computed for single-degree-of-freedom systems with different levels of lateral strength normalized to the strength required to remain elastic when subjected to a relatively large ensemble of recorded earthquake ground motions. Three groups of soil conditions with shear wave velocities higher than 180 m/s are considered. The influence of period of vibration, level of lateral yielding strength, site conditions, earthquake magnitude, distance to the source, and strain-hardening ratio are evaluated and discussed. Mean inelastic displacement ratios and those associated with various percentiles are presented. A special emphasis is given to the dispersion of these ratios. It is concluded that distance to the source has a negligible influence on constant relative strength inelastic displacement ratios. However, for periods smaller than 1 s earthquake magnitude and soil conditions have a moderate influence on these ratios. Strain hardening decreases maximum inelastic displacement at a fairly constant rate depending on the level of relative strength for periods of vibration longer than about 1.0 s while it decreases maximum inelastic displacement non-linearly as the period of vibration shortens and as the relative-strength ratio increases for periods of vibration shorter than 1.0 s. Finally, results from non-linear regression analyses are presented that provide a simplified expression to be used to approximate mean inelastic displacement ratios during the evaluation of existing structures built on firm sites. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: inelastic displacement ratios; evaluation of existing structures; displacement-based design; soils conditions; displacement demands: single-degree-of-freedom systems

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1. INTRODUCTION

Recently introduced displacement-based seismic design criteria use displacements rather than forces as basic demand parameters for the design, evaluation and rehabilitation of structures. However, implementation of displacement-based seismic design criteria into structural engineering practice requires simplified analysis procedures to estimate inelastic displacement demands on structures for ground motions in which the structure is expected to behave non-linearly. This is particularly true when the design is based on design spectra rather than on acceleration time histories. Recent recommendations for the evaluation and rehabilitation of existing structures have introduced simplified analysis methods in which single-degree-of-freedom (SDOF) systems are used to estimate global inelastic displacement demands on structures [1–3]. Furthermore, in these new resource documents global inelastic displacement demands of structures are computed taking into account the relationship between the maximum inelastic displacement demands of non-linear SDOF systems and the maximum elastic displacement demands of linear elastic SDOF systems. Thus, recently there has been a renewed interest on approximate methods to estimate maximum displacement demands of inelastic SDOF systems.

The first study that investigated the relationship between the maximum deformations of inelastic and of elastic systems was conducted by Veletsos and Newmark [4, 5] who studied SDOF systems with an elasto-plastic hysteretic behaviour subjected to simple pulses and to three recorded earthquake ground motions. They observed that in the low frequency region the maximum deformation of the inelastic and elastic systems was approximately the same. This observation gave rise to the well-known equal displacement rule. The study also concluded that, in the high frequency region, the inelastic displacements are significantly higher than their elastic counterparts. In a later study [6], the equal displacement rule was also recommended for the medium frequency region. They also noted that the width of each frequency region is generally different for different ground motions. Shimazaki and Sozen [7] noticed that, in the short period region the ratio of maximum inelastic displacement to maximum elastic displacement depended critically on the lateral strength of the structure relative to the elastic strength demand and that the estimate of the inelastic displacement demand was beyond a simple procedure. These conclusions were also more recently confirmed by Qi and Moehle [8].

Miranda [9, 10] studied ratios of maximum inelastic displacement to maximum elastic displacement of SDOF systems undergoing specific levels of displacement ductility when subjected to 124 earthquake ground motions recorded on different site conditions. Mean constant-ductility ratios of maximum inelastic to maximum elastic response for three types of soil conditions were computed as part of the investigation. The study gave a special insight to this ratio in the short period range and to the limiting periods of the spectral regions where the equal displacement rule is applicable. The results of Miranda were confirmed by Krawinkler and his co-workers at Stanford University [11, 12] using smaller sets of ground motions.

In order to evaluate the non-linear static procedure of the FEMA guidelines for the seismic rehabilitation of buildings [2] Whittaker *et al.* [13] also conducted a study of the ratio of inelastic to elastic displacements. Results of the ratio of mean inelastic displacements to mean elastic displacements were presented corresponding to 20 horizontal components of 10 ground motions recorded on either stiff soil or soft rock sites. The study concluded that for periods smaller than about 1s mean inelastic displacements exceed mean elastic displacements. Furthermore, it was concluded that for systems with lateral strengths smaller than 20% of the strength required to maintain the system elastic, mean inelastic displacements systematically exceed mean elastic displacements, suggesting that the equal displacement rule may not be applicable in these situations.

More recently, Miranda [14] presented a comprehensive statistical study of constant ductility inelastic displacement ratios for the design of structures on firm sites. This study provided new information regarding the dispersion of this ratio and regarding the influence of period, level of inelastic deformation, magnitude, distance to the source and local site conditions. Miranda concluded that for SDOF systems undergoing the same displacement ductility ratio, inelastic displacement ratios were not affected by magnitude or by distance to the source. Furthermore, the study concluded that for sites with average shear wave velocities higher than 180 m/s (600 ft/s) in the upper 30 m (100 ft) of the site profile, local site conditions do not affect significantly constant-ductility inelastic displacement ratios. As part of the study, a simplified equation to estimate ratios of maximum inelastic to maximum elastic displacement as a function of period of vibration and of displacement ductility ratio was also developed.

The simplified equation developed by Miranda [14] to estimate constant-ductility inelastic displacement ratios is very useful in the preliminary design of new structures where control of maximum inelastic deformations is desired for structures where an estimate of the global displacement ductility capacity is known. However, in the evaluation of existing structures the main interest is to determine the global and local deformations that a structure with known lateral strength may undergo when subjected to earthquakes of different intensities. As shown in the next section, the use of constant-ductility inelastic displacement ratios underestimates the expected value of the maximum deformations in systems with known lateral strength. Hence, inelastic displacement ratios corresponding to systems with equal relative lateral strength (lateral yielding strength relative to the lateral strength required to maintain the system elastic) are particularly useful when evaluating existing structures.

While the studies of Shimazaki and Sozen [7] and Whittaker *et al.* [13] provided valuable information regarding constant relative strength inelastic displacement ratios of SDOF systems, none of the two provided expressions of the ratio of maximum inelastic displacement to elastic displacement that could be used during the evaluation of existing structures nor provided information on the dispersion of this ratio. Furthermore, none of the two studies investigated the effect of earthquake magnitude, distance to the source or local site conditions.

More recently Chopra and Goel [15, 16] and Fajfar [17, 18] proposed to estimate the maximum inelastic displacement of existing structures with known lateral strength, by multiplying the yielding displacement by a displacement ductility ratio, μ, computed from existing R_μ–μ–T relations, which typically provide an estimate of the mean strength reduction factor R_μ as a function of the displacement ductility ratio and the period of vibration T. While this approach is very simple and provides a way to estimate the maximum inelastic displacement, Miranda [19] has shown that the ductility demand computed from R_μ–μ–T relations does not correspond to the mean displacement ductility demand of system with relative strength equal to R_μ, and that the computed displacement ductility ratio is the first approximation of the mean displacement ductility demand, hence this procedure introduces a systematic error that will tend to underestimate the maximum inelastic displacement. The error will typically increase with increasing ductility, although the actual size of the error depends on the particular R_μ–μ–T relation that is used. For the evaluation of existing structures more accurate estimates of maximum inelastic displacements of systems where the relative strength is known can be achieved by using directly results from statistical studies.

The objective of this paper is to present the results of a statistical study of the ratio of maximum inelastic displacement demand to maximum elastic displacement demand for SDOF systems on firm sites with known relative strength. The effects of period of vibration, level of lateral strength, earthquake magnitude, distance to the source and local site conditions are investigated. The dispersion of constant relative strength inelastic displacement ratios is assessed. This study makes use of improved information that has been made available recently on the geological characteristics at accelerographic recording stations in California. The investigation is limited to rock and relatively firm soil sites with shear wave velocities higher than 180 m/s in the upper 30 m of the site profile.

2. INELASTIC DISPLACEMENT RATIOS

The inelastic displacement ratio, C_R, is defined as the maximum lateral inelastic displacement demand, $\Delta_{\text{inelastic}}$, divided by the maximum lateral elastic displacement demand, Δ_{elastic}, on systems with the same mass and initial stiffness (i.e. same period of vibration) when subjected to the same earthquake ground motion. In both cases displacements are relative to the ground. Mathematically this is expressed as

$$C_R = \frac{\Delta_{\text{inelastic}}}{\Delta_{\text{elastic}}} \tag{1}$$

In Equation (1) the $\Delta_{\text{inelastic}}$ is computed in systems with constant yielding strength relative to the strength required to maintain the system elastic (i.e. constant relative strength). Here the relative lateral strength is measured by the strength ratio R, which is defined as

$$R = \frac{mS_{\text{a}}}{F_{\text{y}}} \tag{2}$$

where m is the mass of the system, S_{a} is the acceleration spectral ordinate and F_{y} is the lateral yielding strength of the system. The numerator in Equation (2) represents the lateral strength required to maintain the system elastic, which sometimes is also referred to as the elastic strength demand.

The nomenclature in Equation (1) is meant to be consistent with the nomenclature used in NEHRP publications [2, 3] in which the letter C is used as a factor modifying elastic displacements and is also consistent with the nomenclature previously used by Miranda [14] in which the subscript in the inelastic displacement ratio represents the parameter that remains constant. Thus, constant ductility inelastic displacement ratios are represented by C_μ, and constant relative strength (or constant strength ratio) inelastic displacement ratios are represented by C_R. Both types of inelastic displacement ratios permit the estimation of maximum inelastic displacement demands from maximum elastic displacement demands.

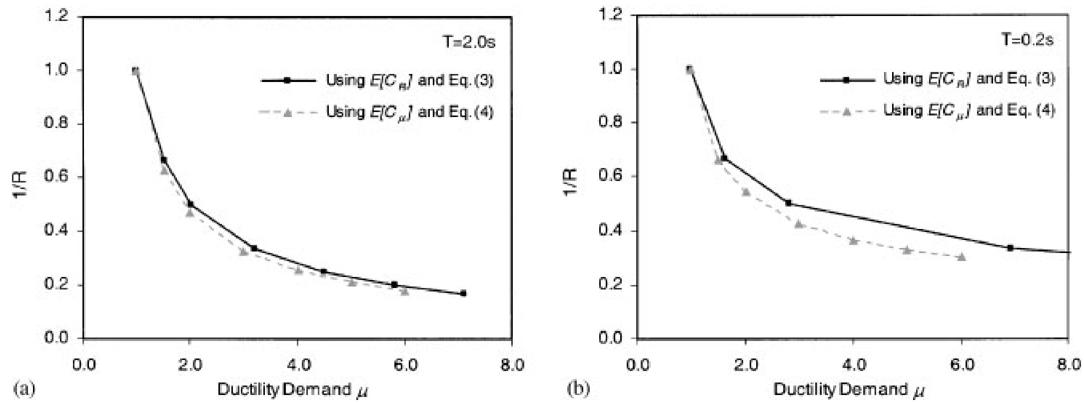


Figure 1. Relationship between normalized strength ($1/R$) and displacement ductility demand. (a) For a $T = 2.0$ s system, (b) for a $T = 0.2$ s system.

Inelastic displacement ratios were computed for SDOF systems having a viscous damping ratio of 5%, a non-linear elasto-plastic hysteretic behaviour, and with the following strength ratios $R = 1$, 1.5, 2, 3, 4, 5 and 6. For each earthquake record and each relative strength, the inelastic displacement ratios were computed for a set of 50 periods of vibration between 0.05 and 3.0 s. Unlike the constant ductility inelastic displacement ratio C_μ that has to be computed through iteration on the lateral strength until the computed displacement ductility demand is within a certain tolerance equal to the target ductility ratio, the constant relative strength inelastic displacement ratio C_R can be computed without any iteration and thus, for a given acceleration time history, it is significantly faster to compute.

Figure 1 shows the relationship between the lateral strength of SDOF systems with a period of vibration of 1.0 s, and the maximum displacement when subjected to 264 earthquake ground motions recorded on firm sites [14]. In this figure, the lateral strength is normalized by the lateral strength required to maintain the system elastic (i.e. $F_y/(mS_a) = 1/R$), and the maximum deformation is normalized by the yield displacement. The continuous line represents the relationship between the lateral strength and the expected value of the ductility demand computed using the expected value of C_R as follows:

$$E[\mu] = R \cdot E[C_R] \tag{3}$$

where μ is the displacement ductility ratio and $E[\ ]$ denotes expectation. The dotted line represents the relationship between the displacement ductility ratio and the expected value of the normalized lateral strength computed for $\mu = 1$, 1.5, 2, 3, 4, 5 and 6 using the expected value of C_μ as follows:

$$E\left[\frac{1}{R}\right] = \frac{E[C_\mu]}{\mu} \tag{4}$$

It can be seen that, in this case, the use of $E[C_\mu]$ using Equation (4) leads to very similar results as when using $E[C_R]$ in Equation (3). However, it can be noted that for a given R the ductility demand computed with constant ductility inelastic displacement ratios is always smaller than the one computed with constant relative strength inelastic displacement ratios. Furthermore, the difference increases as R and μ increase. The practical implication of this observation is that the use of constant ductility inelastic displacement ratios like those reported by Miranda [14] if used for the evaluation of existing structures, in which R is known for a given ground motion, can lead to underestimations of the maximum inelastic displacement demands. The underestimation increases as the dispersion on C_μ and C_R increases. For short period structures, where the dispersion on C_R is large, the underestimation can be much larger. An example similar to that shown in Figure 1(a) but for a period of 0.2 s is shown in Figure 1(b). In this case, for $R=3$ the underestimation in maximum displacement can be larger than 40%. Hence it is clear that for the evaluation of existing structures there is a need for statistical studies on C_R. Only when there is no dispersion on C_μ and C_R, as for example in the case of $R=1$ or for a single ground motion (for any level of R) do Equations (3) and (4) lead to the same displacement. For a further discussion on the reason for this difference the reader is referred to Miranda [19].

3. EARTHQUAKE GROUND MOTIONS USED IN THE STUDY

A total of 216 earthquake acceleration time histories recorded in the state of California in 12 different earthquakes with magnitude ranging from 5.8 to 7.7 were used in this study. A particularly large number of earthquake ground motions was selected in order to assess the dispersion of the inelastic displacement ratios and in order to be able to obtain inelastic displacement ratios corresponding to different percentiles. All the ground motions selected have the following characteristics: (i) recorded on accelerographic stations where enough information exists on the geological and geotechnical conditions at the site that enables the classification of the recording site in accordance to recent code recommendations [2, 3, 20]; (ii) recorded on rock or firm sites with average shear wave velocities higher than 180 m/s (600 ft/s) in the upper 30 m (100 ft) of the site profile; (iii) recorded on free field stations or in the first floor of low-rise buildings with negligible soil–structure interaction effects; (iv) recorded in earthquakes with surface wave magnitudes (M_s) larger than 5.7; and (v) records in which at least one of the two horizontal components had a peak ground acceleration larger than 40 cm/s^2.

The earthquake ground motions were divided into three groups according to the local site conditions at the recording station. The first group consisted of 72 ground motions recorded on stations located on rock with average shear wave velocities between 760 m/s (2500 ft/s) and 1525 m/s (5000 ft/s). The second group consisted of 72 records obtained on stations on very dense soil or soft rock with average shear wave velocities between 360 m/s (1200 ft/s) and 760 m/s while the third group consisted of 72 ground motions recorded on stations on stiff soil with average shear wave velocities between 180 m/s (600 ft/s) and 360 m/s. Recording stations in the first group correspond to site class B according to recent design provisions [2, 3, 20] while recording stations in the second and third groups correspond to site classes C and D, respectively. For a complete list of all ground motions including peak ground accelerations, earthquake magnitude, site class at the recording station, and distance to the horizontal projection of the fault rupture see Tables I–III.

Table I. Earthquake ground motions recorded on NEHRP site class B used in this study.

Earthquake name	Magnitude (Ms)	Station name	Station number	Distance (km)	Comp. 1 (deg)	PGA (cm/s^2)	PGV (cm/s)	Comp. 2 (deg)	PGA (cm/s^2)	PGV (cm/s)
1971 San Fernando	6.5	Lake Hughes, Array Station 4	126	19.6	111	168.2	5.7	201	143.5	7.1
1971 San Fernando	6.5	Lake Hughes, Array Station 9	127	23.0	21	119.3	4.5	291	109.4	3.9
1979 Imperial Valley	6.8	Superstition Mountain	286	26.0	135	189.2	6.3	45	108.0	5.1
1984 Morgan Hill	6.1	Gilroy 1, Gavillan Coll.	47 379	16.0	230	57.5	2.9	320	93.4	2.9
1986 Palm Springs	6.0	Silent Valley, Poppet Flat	12 206	23.7	0	102.6	3.9	90	107.4	4.0
1986 Palm Springs	6.0	Winchester, Hidden Valley Farms	13 200	49.8	90	54.6	1.3	270	56.5	1.3
1986 Palm Springs	6.0	Winchester, Bergman Ranch	13 199	55.3	0	62.2	1.9	90	85.7	1.8
1986 Palm Springs	6.0	Murrieta Hot Springs, Colling Ranch	13 198	61.0	0	45.9	1.8	90	49.4	1.3
1986 Palm Springs	6.0	Whitewater Trout Farm	5072	7.3	180	482.7	34.7	270	600.4	31.5
1986 Palm Springs	6.0	Anza Red Mountain	5224	45.6	270	102.0	5.2	360	126.5	3.4
1987 Whittier	6.1	Mt Wilson, CIT Seismic Station	24 399	22.1	0	121.3	3.3	90	171.3	4.6
1987 Whittier	6.1	Los Angeles, Gritfith Park Observatory	141	22.3	0	133.8	3.6	360	121.4	4.1
1989 Loma Prieta	7.1	Gilroy 1, Gavillan Coll.	47 379	10.5	90	433.6	33.9	360	426.6	31.6
1989 Loma Prieta	7.1	Hollister, SAGO south cinega road surface	47 189	32.4	261	70.7	10.3	351	65.3	9.3
1989 Loma Prieta	7.1	Monterey, City Hall	47 377	42.7	90	61.1	5.8	360	68.5	3.5
1989 Loma Prieta	7.1	South San Francisco, Sierra Point	58 539	67.6	115	57.2	7.1	205	102.7	8.8
1989 Loma Prieta	7.1	San Francisco, Dimond Heights	58 130	75.9	90	110.8	14.3	360	96.4	10.5
1989 Loma Prieta	7.1	Piedmont, Piedmont Jr. High Grounds	58 338	77.2	45	81.2	8.2	315	69.6	9.1
1989 Loma Prieta	7.1	San Francisco, Rincon Hill	58 151	78.5	90	88.5	10.4	0	78.6	6.7
1989 Loma Prieta	7.1	San Francisco, Pacific Heights	58 131	80.5	270	60.2	12.8	360	46.3	9.2
1989 Loma Prieta	7.1	San Francisco, Cliff House	58 132	87.4	0	73.1	11.2	90	105.7	21.0
1989 Loma Prieta	7.1	San Francisco, Telegraph Hill	58 133	88.0	90	51.2	5.5	0	90.5	9.5
1989 Loma Prieta	7.1	Point Bonita	58 043	88.1	297	71.4	12.9	207	69.9	11.4
1991 Sierra Madre	5.8	Mt Wilson, CIT Seismic Station	24 399	5.3	0	270.7	13.0	90	196.2	7.5
1992 Landers	7.5	Twentynine Palms Park Maintenance Bldg	22 161	41.9	0	78.7	3.7	90	59.1	4.9
1992 Landers	7.5	Silent Valley, Poppet Flat	12 206	51.3	90	39.4	5.1	0	48.9	3.8
1992 Landers	7.5	Amboy	21 081	68.3	0	88.3	11.0	90	143.2	20.0
1994 Northridge	6.8	Malibu Canyon, Griffith Observatory	5080	20.2	360	176.4	12.3	270	270.0	17.7
1994 Northridge	6.8	Lake Hughes, Array Station 9	24 272	28.4	90	221.2	10.1	360	154.5	8.4
1994 Northridge	6.8	Los Angeles, Temple & Hope	24 611	32.2	180	189.1	20.0	90	123.7	13.9
1994 Northridge	6.8	Lake Hughes, Array Station 4	24 469	34.0	0	56.4	5.1	90	82.4	5.1
1994 Northridge	6.8	Mt Wilson, CIT Seismic Station	24 399	36.9	90	130.7	5.8	360	228.5	7.4
1994 Northridge	6.8	Los Angeles, City Terrace	24 592	37.1	90	258.0	12.8	0	310.1	14.1
1994 Northridge	6.8	Antelope Buttes	24 310	48.6	90	99.7	4.3	0	44.9	3.6
1994 Northridge	6.8	San Pedro, Palos Verdes	14 159	58.5	90	93.1	6.6	0	98.9	5.6
1994 Northridge	6.8	Leona Valley #3	24 307	37.8	0	82.4	8.5	90	104.0	8.1

Table II. Earthquake ground motions recorded on NEHRP site class C used in this study.

Earthquake name	Magnitude (Ms)	Station name	Station number	Distance (km)	Comp. 1 (deg)	PGA (cm/s^2)	PGV (cm/s)	Comp. 2 (deg)	PGA (cm/s^2)	PGV (cm/s)
1952 Kern County	7.7	Santa Barbara, Courthouse	283	85.0	42	87.8	12.1	132	128.6	15.5
1952 Kern County	7.7	Pasadena, CIT Athenaeum	475	109.0	180	46.5	5.6	270	52.1	9.2
1971 San Fernando	6.5	Lake Hughes, Array Station 12	128	17.0	21	346.2	17.0	291	277.9	12.7
1971 San Fernando	6.5	Glendale, 633 E, Broadway	122	18.0	110	265.7	10.3	200	209.1	13.4
1971 San Fernando	6.5	Lake Hughes #1, Fire Station #78	125	25.0	21	145.5	18.0	111	108.9	11.7
1971 San Fernando	6.5	Castaic Old Ridge Route	110	26.0	21	309.4	15.6	291	265.4	25.9
1971 San Fernando	6.5	Pearblossom Pump Plant	585	36.0	0	91.5	4.7	270	120.5	5.6
1979 Imperial Valley	6.8	El Centro, Parachute Test Facillity	5051	14.0	225	106.9	17.8	315	200.2	16.1
1984 Morgan Hill	6.1	Gilroy #6, San Ysidro Microwave Site	57 383	11.5	0	214.8	11.4	90	280.4	36.7
1984 Morgan Hill	6.1	Gilroy Gavillan College Phys Sch. Bldg	47 006	16.0	337	85.9	2.9	67	95.0	3.6
1987 Whittier	6.1	Garvey Reservoir Abutment Bldg	709	3.4	60	367.1	15.8	330	468.2	19.0
1987 Whittier	6.1	Alhambra, 900 S. Fremont	24 461	6.7	180	286.2	22.0	270	374.3	16.3
1987 Whittier	6.1	San Marino, SW Academy	24 401	7.7	270	136.5	4.8	360	183.8	12.9
1987 Whittier	6.1	Inglewood, Union Oil Yard	14 196	22.5	0	246.1	18.1	90	219.3	8.9
1987 Whittier	6.1	Long Beach, Recreation Park	14 241	29.6	90	57.2	3.5	180	53.8	5.5
1987 Whittier	6.1	Sylmar, Olive View Medical Center	24 514	45.9	0	55.8	4.0	90	50.3	3.5
1987 Whittier	6.1	Riverside, Airport	13 123	57.8	180	38.4	1.4	270	56.8	1.4
1987 Whittier	6.1	Lancaster, Medical Office Bldg FF	24 526	72.2	10	59.4	2.9	100	59.6	3.0
1987 Whittier	6.1	Castaic, Old Ridge Route	24 278	77.3	0	67.2	4.4	90	65.4	4.5
1989 Loma Prieta	7.1	Corralitos, Eureka Canyon Road	57 007	0.0	90	469.4	47.5	360	617.7	55.2
1989 Loma Prieta	7.1	Gilroy, Gavillan College Phys Sch Bldg	47 006	10.9	67	349.1	28.9	337	310.0	23.0
1989 Loma Prieta	7.1	Saratoga, Aloha Ave.	58 065	12.4	90	316.2	43.5	0	494.5	41.3
1989 Loma Prieta	7.1	Santa Cruz, UCSC	58 135	12.5	90	401.5	21.2	360	433.1	21.2
1989 Loma Prieta	7.1	Gilroy 6, San Ysidro Microwave site	57 383	19.9	90	166.9	14.2	0	112.2	12.8
1989 Loma Prieta	7.1	Coyote Lake Dam, downstream	57 504	21.7	285	174.7	22.6	195	154.7	13.0
1989 Loma Prieta	7.1	Woodside, Fire Station	58 127	38.7	90	79.7	14.7	0	79.5	15.5
1989 Loma Prieta	7.1	Fremont, Mission San Jose	57 064	42.6	90	100.5	8.8	0	117.7	11.5
1989 Loma Prieta	7.1	Hayward, CSUH Stadium	58 219	56.7	90	82.6	6.4	0	72.5	5.6
1989 Loma Prieta	7.1	Berkeley, Lawrence Berkeley Lab.	58 471	83.9	90	114.4	20.9	0	47.7	9.2
1992 Landers	7.5	Joshua Tree, Fire Station	22 170	7.1	0	268.3	27.5	90	278.4	43.2
1994 Northridge	6.8	Castaic Old Ridge Route	24 278	24.6	360	504.2	52.2	90	557.1	52.1
1994 Northridge	6.8	San Marino, SW Academy	24 401	35.5	360	148.2	5.4	90	122.5	7.3
1994 Northridge	6.8	Alhambra, 900 S. Fremont	24 461	37.2	360	78.3	12.7	90	99.1	9.7
1994 Northridge	6.8	Lake Hughes #1, Fire Station #78	24 271	37.7	0	84.9	9.2	90	75.2	9.4
1994 Northridge	6.8	Littlerock, Brainard Canyon	23 595	47.9	90	70.6	5.0	180	59.0	5.3
1994 Northridge	6.8	Rancho Palos Verdes, Hawthorne Blvd.	14 404	53.8	0	71.1	5.0	90	52.7	3.3

Table III. Earthquake ground motions recorded on NEHRP site class D used in this study.

Earthquake name	Magnitude (Ms)	Station name	Station number	Distance (km)	Comp. 1 (deg)	PGA (cm/s^2)	PGV (cm/s)	Comp. 2 (deg)	PGA (cm/s^2)	PGV (cm/s)
1952 Kern County	7.7	Los Angeles, Hollywood Storage PE Lot	135	107.0	90	41.2	6.0	180	58.1	5.3
1968 Borrego Mtn	6.7	El Centro, Imperial Valley Irrigation District	117	45.0	180	127.8	26.3	270	56.3	13.2
1971 San Fernando	6.5	Los Angeles, Hollywood Storage Bldg	135	23.0	90	207.0	18.9	180	167.3	14.9
1971 San Fernando	6.5	Vernon, Cmd Terminal	288	33.5	187	80.5	6.4	277	104.6	9.8
1971 San Fernando	6.5	Santa Ana, Engineering Bldg	281	71.5	176	26.8	2.9	266	28.2	3.1
1979 Imperial Valley	6.8	Calexico, Fire Station	5053	10.6	225	269.6	21.2	315	196.9	16.0
1979 Imperial Valley	6.8	El Centro #11, McCabe Union School	5058	12.6	140	355.4	24.7	230	374.5	29.8
1979 Imperial Valley	6.8	El Centro #3, Pine Union School	5057	12.7	140	261.7	46.8	230	218.1	39.9
1979 Imperial Valley	6.8	El Centro #12, 907 Brockman Road	931	18.0	140	138.7	11.1	230	113.4	10.7
1979 Imperial Valley	6.8	Plaster City, Storehouse	5052	32.0	135	55.5	4.2	45	41.9	3.8
1979 Imperial Valley	6.8	Coachella, Canal #4	5066	49.0	45	113.6	12.5	135	125.7	15.6
1984 Morgan Hill	6.1	Gilroy #2, Hwy 101/Bolsa Road Motel	47 380	1.0	0	153.7	5.1	90	210.0	12.6
1984 Morgan Hill	6.1	Gilroy #7, Mantnilli Ranch, Jamison Rd	57 425	13.7	0	183.0	7.4	90	111.5	6.0
1984 Morgan Hill	6.1	Gilroy #3, Sewage Treatment Plant	47 381	14.4	0	177.0	11.2	90	189.8	12.7
1987 Whittier	6.1	Bell Los Angeles Bulk Mail Center	5129	10.6	10	322.1	31.1	280	436.9	39.7
1987 Whittier	6.1	Vernon, Cmd Terminal	288	11.1	7	267.3	25.4	277	239.9	22.9
1987 Whittier	6.1	Downey, County Maintenance Bldg	14 368	16.2	180	193.2	28.8	270	150.7	13.4
1987 Whittier	6.1	Los Angeles, Hollywood Storage Bldg	24 303	23.8	0	201.3	9.0	90	103.7	6.9
1987 Whittier	6.1	Century City, LA Country Club South	24 390	29.6	0	57.6	3.7	90	67.2	4.2
1987 Whittier	6.1	Pomona, 4th and Locust FF	23 525	29.9	12	68.4	2.4	102	49.0	2.3
1987 Whittier	6.1	Long Beach, Harbor Administration Bldg	14 395	32.8	0	48.2	5.5	90	68.9	3.6
1987 Whittier	6.1	Rancho Cucamonga, Law and Justice Center	23 497	45.5	90	55.5	1.4	360	45.3	1.4
1987 Whittier	6.1	Arleta, Nordhoff Avenue Fire Station	24 087	45.7	180	87.1	4.8	270	84.2	5.7
1987 Whittier	6.1	Rosamond, Goode Ranch	24 274	89.0	0	73.8	3.5	90	50.4	3.1
1989 Loma Prieta	7.1	Gilroy 2, Hwy 101 Bolsa Road Motel	47 380	12.1	90	316.3	39.1	0	394.2	32.9
1989 Loma Prieta	7.1	Gilroy 3, Sewage Treatment Plant	47 381	14.0	90	362.0	44.7	0	531.7	35.7
1989 Loma Prieta	7.1	Agnews, Agnews State Hospital	57 066	27.0	90	157.6	17.6	0	163.1	26.0
1989 Loma Prieta	7.1	Hayward, John Muir School	58 393	58.9	90	136.0	11.5	0	166.5	13.7
1989 Loma Prieta	7.1	Oakland, 2 story	58 224	76.3	290	238.3	36.1	200	187.3	19.9
1989 Loma Prieta	7.1	Richmond, City Hall parking lot	58 505	92.7	280	103.6	14.2	190	122.7	17.3
1992 Landers	7.5	Yermo, Fire Station	22 074	26.3	270	240.0	51.5	360	148.6	29.7
1992 Landers	7.5	Palm Springs, Airport	12 025	28.2	0	74.2	10.9	90	87.2	13.8
1992 Landers	7.5	Fort Irwin	24 577	65.5	0	111.4	9.7	90	119.8	16.4
1992 Landers	7.5	Baker, Fire Station	32 075	88.3	50	105.6	9.4	140	103.6	11.0
1992 Landers	7.5	Pomona, 4th and Locust FF	23 525	117.6	0	65.5	12.3	90	43.2	8.5
1994 Northridge	6.8	Los Angeles, Hollywood Storage Bldg	24 303	24.8	360	381.4	22.3	90	227.0	18.1

4. STATISTICAL RESULTS

4.1. Mean ratios for different local site conditions

A total of 64,800 inelastic displacement ratios were computed as part of this study (corresponding to 216 ground motions, 50 periods of vibration and 6 levels of relative strength). Mean inelastic displacement ratios were then computed by averaging results for each period, each strength ratio and each group of local site conditions at the recording station. For the site classes considered here current seismic design provisions in the US [3] specify linear elastic design spectra that are significantly different from each other. Thus, it is particularly important to know if inelastic displacement ratios to be used for estimating maximum inelastic displacements from maximum elastic displacements while evaluating existing structures are affected by local site conditions, and if so, to quantify these differences.

Figure 2 shows mean inelastic displacement ratios corresponding to the three groups of site conditions considered here. It can be seen that, in general, constant relative strength exhibits the same trend regardless of the local site conditions. These ratios are characterized by being larger than 1 in the short period spectral region (maximum inelastic displacements larger than maximum elastic displacements) and relatively close to 1 (maximum inelastic displacements *on average* approximately equal to maximum elastic displacement) for long periods. For periods smaller than 1.0 s inelastic displacement ratios are strongly dependent on the period of vibration and on the lateral strength ratio. In general, in this spectral region maximum inelastic displacements become much larger than maximum elastic displacements as the lateral strength ratio increases (i.e. as the lateral strength decreases with respect to the lateral strength required to maintain the system elastic) and as the period of vibration decreases. Furthermore, constant relative strength inelastic displacement ratios tend towards ∞ as the period of vibration tends to zero, which means that existing structures with very short periods may undergo very large inelastic displacement demands relative to their elastic counterparts unless they have lateral strengths that allow them to remain elastic or nearly elastic. It is important to notice that the limiting period dividing spectral regions where the equal displacement rule is applicable from those where this rule is not applicable and is unconservative (and produces an underestimation of the maximum lateral displacement demand) depends primarily on the lateral strength ratio, although it is also influenced by local site conditions. In general, this limiting period increases as lateral strength ratio increases and as the average shear wave velocity in the upper 30 m of the site profile decreases (i.e. as site conditions become softer). For example, for structures on a site class C the equal displacement rule is applicable on average for periods longer than about 0.4 s for a lateral strength ratio of 1.5 but the rule is only approximately correct for periods longer than about 1.0 s for lateral strength ratios of 6. Similarly, for a lateral strength ratio of 2, the equal displacement rule is approximately correct for periods longer than about 0.45, 0.65 and 0.8 s for structures on site classes B, C and D, respectively. It can be seen that in the short period spectral region C_R increases as the average shear wave velocity in the upper 30 m of the site profile decreases.

4.2. Mean and median ratios for all site classes

Figure 3 shows mean constant-ductility inelastic displacement ratios corresponding to all 216 ground motions, regardless of the site conditions at the recording station. Ratios shown in

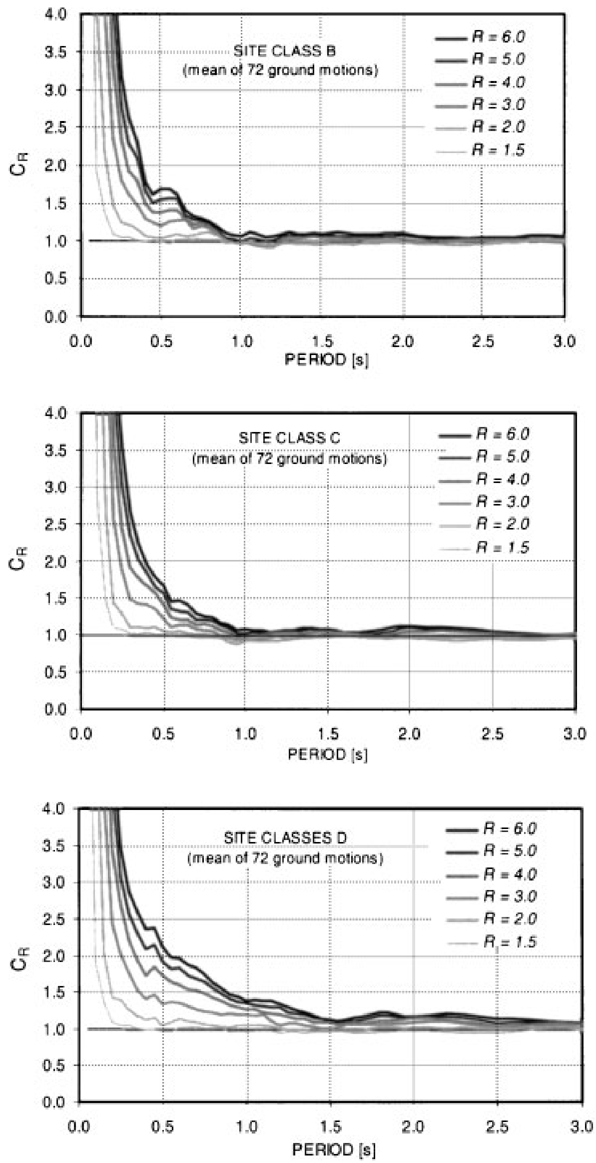


Figure 2. Mean inelastic displacement ratios for NEHRP site classes B, C and D.

this figure represent what, on average, can be expected for existing structures built on firm sites. Again, it can be seen that in the short period region the ratio of inelastic to elastic displacement demand is strongly dependent on the relative strength of the system. Furthermore, in this period region the equal displacement rule can result in significant underestimations of the maximum inelastic displacement demand.

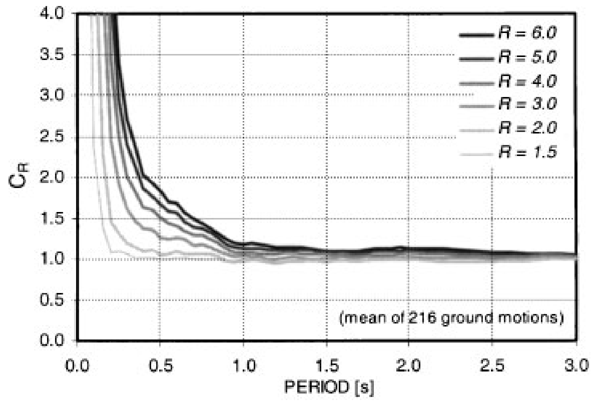


Figure 3. Mean inelastic displacement ratios for all 216 ground motions recorded in NEHRP site classes B, C and D.

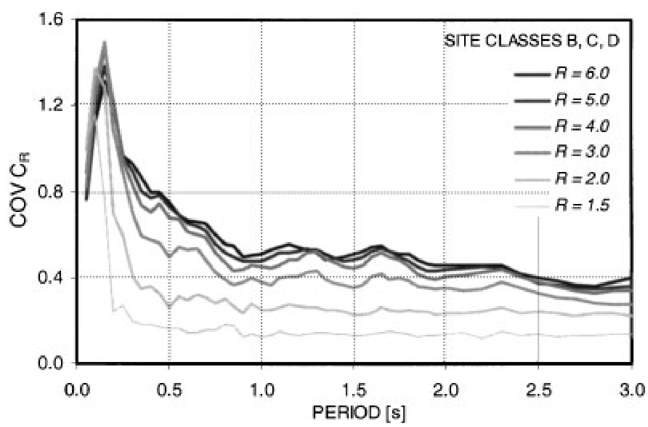


Figure 4. Coefficients of variation of inelastic displacement ratios for all 216 ground motions recorded in NEHRP site classes B, C and D.

4.3. Dispersion

While mean inelastic displacement ratios are very important, as they represent what can be expected on average, it is equally important to quantify the level of dispersion in C_R. A common and effective way to quantify the dispersion is through the coefficient of variation (COV), which is defined as the ratio of the standard deviation to the mean. Figure 4 shows COVs of inelastic displacement ratios corresponding to ground motions from all site classes considered herein. It can be seen that, as expected, dispersion increases as the level of inelastic deformation increases. Dispersion is particularly high for periods of vibration ($T < 0.4$ s) regardless of the lateral strength ratio. Relatively high dispersion also occurs for high values of R and periods up to 1.5 s. Furthermore, with the exception of very short periods (smaller than 0.15 s), for a given level of ductility demand the COV decreases with increasing periods.

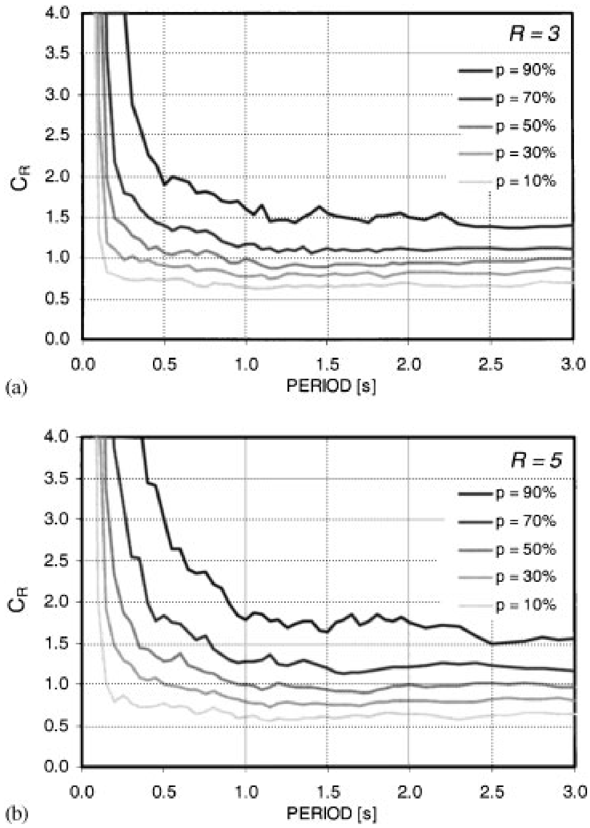


Figure 5. Inelastic displacement ratios corresponding to different percentiles for (a) $R=3$ and (b) $R=5$.

This trend is more noticeable for systems with smaller relative strengths (i.e. with higher values of R). In general, dispersion in C_R is larger than the dispersion reported by Miranda [14] for C_μ, particularly for periods smaller than 1.0 s.

Another way to consider the dispersion on C_R is to compute inelastic displacement ratios corresponding to different percentiles. Inelastic displacement ratios for a lateral strength ratio of 3 corresponding to percentiles of 10%, 30%, 50%, 70% and 90% are shown in Figure 5(a). It can be seen that although median inelastic displacement ratios ($p=50\%$) are approximately equal to one for periods longer than about 1.0 s, there is an 80% probability that inelastic displacement ratios will be between the curves associated to percentiles 10 and 90%, which in this spectral region implies that in 80% of the cases C_R varies approximately between 0.65 and 1.5. Similarly, inelastic displacement ratios in this spectral region in 40% of the cases (between $p=30$ and 70%) are larger than 0.8 and smaller than 1.15. Inelastic displacement ratios corresponding to the same percentiles but for a lateral strength ratio of five are shown in Figure 5(b). In this case it can be seen that for periods of vibration larger than about 1.0s there is an 80% probability that maximum inelastic displacement demand will be approximately between 0.6 and 1.85 times the maximum elastic displacement demand.

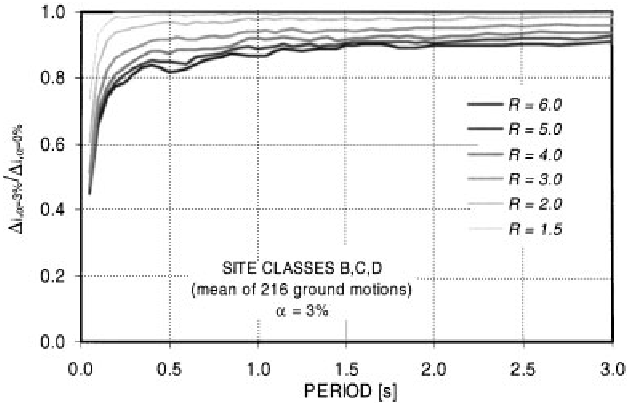


Figure 6. Mean ratios of maximum deformation of bilinear to elastoplastic systems for all 216 ground motions recorded in NEHRP site classes B, C and D.

4.4. Effect of post-yield stiffness

In order to study further the effect of positive post-yield stiffness on the maximum inelastic displacement demands, maximum deformations of bilinear systems with post-yield stiffness ratios (post-yield stiffness normalized by initial stiffness) of $\alpha = 3$, 5 and 10% were computed when subjected to all 216 ground motions. Then, ratios of the maximum inelastic deformation of bilinear systems to maximum deformation of the elastoplastic system was obtained for each record, each period of vibration, and each lateral strength ratio. Figure 6 shows mean ratios of maximum inelastic displacement demand of bilinear systems having post-yield stiffness of 3% of the initial stiffness to the maximum inelastic displacement demand of elastoplastic systems. It can be seen that the maximum inelastic deformation of the bilinear systems becomes smaller with respect to the one of the elastoplastic system as the strength ratio increases. For periods of vibrations larger than about 1.0 s this ratio of maximum deformation remains approximately constant. However, for periods smaller than about 0.5 s the maximum deformation of systems with positive post-yield stiffness can be significantly smaller than that of elastoplastic systems. Figure 7 shows the coefficient of variation of inelastic displacement ratio of bilinear systems. Comparing Figures 4 and 7 it can be seen that the dispersion of inelastic displacement ratios of systems with positive post-yield stiffness is essentially the same as that with elastoplastic systems except for periods smaller than about 0.25 s where small reductions in dispersion are observed.

4.5. Effect of soil conditions

Most structures are built on sites that are classified as firm sites (site classes B, C and D). Thus, it is important to quantify the differences on inelastic displacement ratios computed from ground motions recorded in these site classes. In order to assess the effect of local site conditions, that is to evaluate the differences in C_R for different site conditions within firms sites (site classes B, C and D), ratios of mean C_R on each group to mean C_R computed from

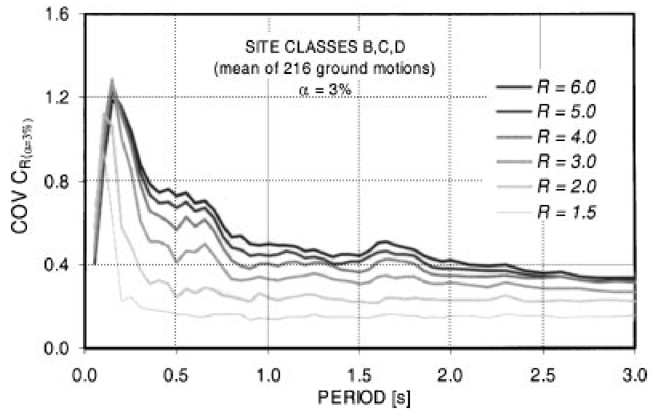


Figure 7. Coefficients of variation of maximum inelastic displacement ratio of bilinear systems with $\alpha = 3\%$ computed with all 216 ground motions recorded in NEHRP site classes B, C and D.

all 216 ground motions were computed. Figures 8(a)–(c) show mean inelastic displacement ratios for site class B, site class C and site class D normalized by mean inelastic displacement of the four site classes, respectively. It can be seen if one neglects the effects of local site conditions for structures on rock (site class B) and uses mean values from all 216 ground motions one would overestimate maximum inelastic displacement demands. For periods between 0.2 and 1.2 s the overestimation is less than 15% while for periods longer than 1.2 s the overestimation is less than 5%. For structures on site class C the use of mean inelastic displacement ratios from all site classes considered here would produce small underestimations of maximum displacements for $T < 0.2$ s, small overestimations for $0.2\text{ s} < T < 0.9$ s and practically no errors on average for $T > 1.0$ s. Meanwhile for structures on sites classified as D ignoring the effects of site conditions in the estimation of C_R could result in small underestimations of maximum displacement for structures with periods smaller than 1.4 s. It can be seen that the difference in inelastic displacement ratio produced by local site conditions increases with increasing lateral strength ratio. Thus, for lateral strength ratios smaller than 3 the errors produced by neglecting the effect of local site conditions on C_R are typically smaller than 10%, while for strength ratios of 4 and 5 site conditions are typically smaller than 20%.

Differences in mean inelastic displacement ratios for these site classes can be slightly reduced if periods of vibration are normalized by a characteristic period for each site class as suggested by Chopra and Chintanapakdee [21]. Figure 9 shows a comparison of mean inelastic displacement ratios for $R = 4$ and 6 for sites classes B, C and D when periods of vibration are normalized by characteristic site periods of 0.75, 0.85 and 1.05 s, respectively. In this case mean inelastic displacement ratios are closer to each other.

4.6. Effect of earthquake magnitude

Elastic spectral ordinates are dependent on the magnitude of the earthquake. Thus, it is important to know to what extent earthquake magnitude affects inelastic displacement ratios. In order

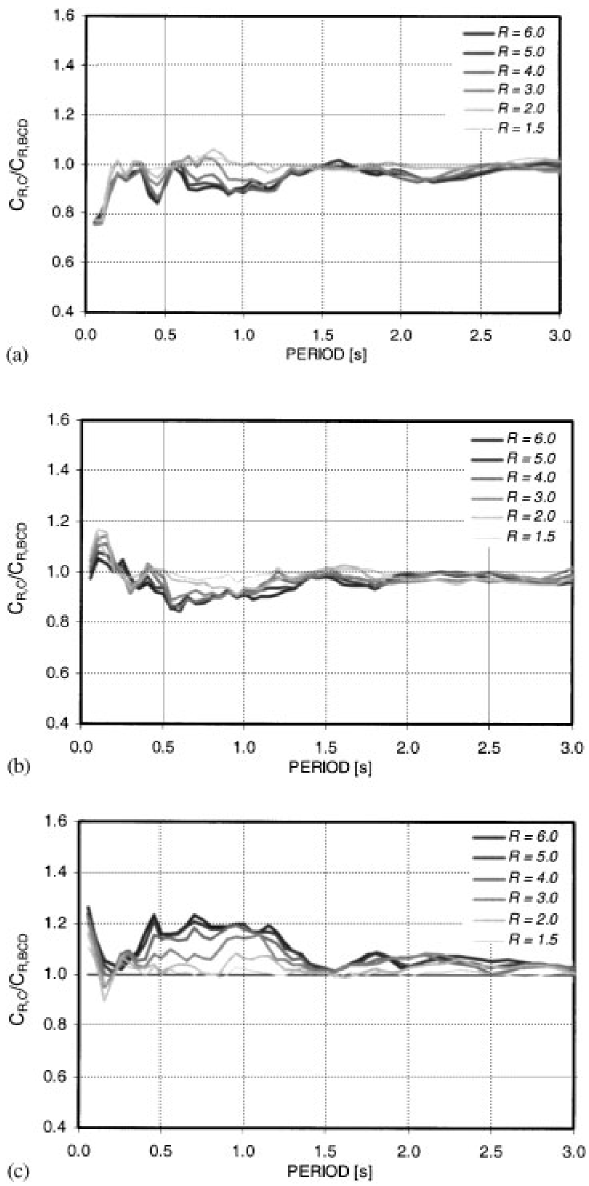
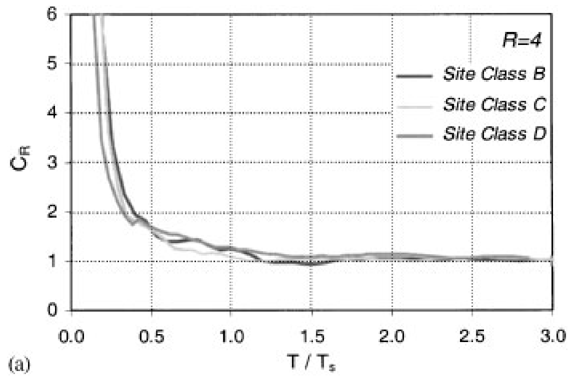
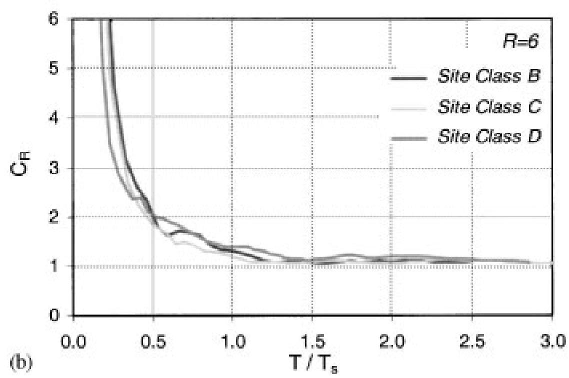


Figure 8. Mean inelastic displacement ratios on each group normalized by mean ratios from all ground motions: (a) site class B; (b) site class C; (c) site class D.

to study the effect of earthquake magnitude, mean inelastic displacement ratios were computed for ground motions recorded on class D sites and then grouped according to the magnitude of the earthquake in which they were recorded. A comparison of mean inelastic displacement ratios computed ground motions in three ranges of magnitudes for lateral strength ratios equal



(a)



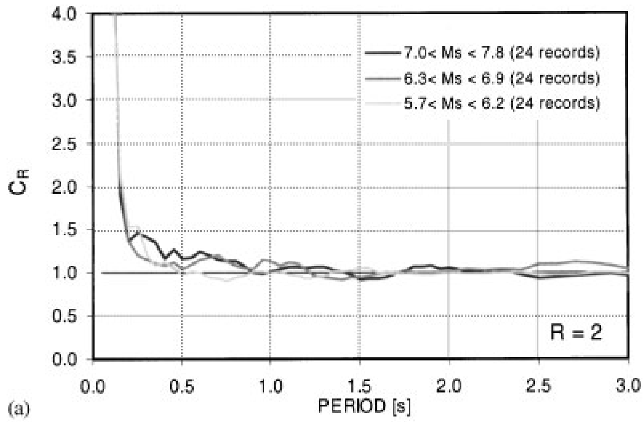
(b)

Figure 9. Mean inelastic displacement ratios normalized by a characteristic period for each site class for (a) $R = 4$ and (b) $R = 6$.

to 2 and 4 is shown in Figure 10. It can be seen that for a lateral strength ratio equal to 2, earthquake magnitude has practically no effect on C_R. However, for weaker structures relative to the intensity of the ground motions (i.e. for higher values of R) magnitude can influence C_R for systems with short periods of vibration. It can be seen that for $R = 4$ the inelastic displacement ratios of ground motions recorded in earthquakes with surface-wave magnitudes between 5.7 and 6.2 tend to be smaller than those of ground motions recorded in earthquakes with higher magnitudes. In particular, for $R = 4$ and $T = 0.5$s, mean values of C_R of ground motions recorded in earthquakes with magnitudes higher than 6.3 are approximately two times higher those from ground motions recorded in earthquake with magnitudes between 5.7 and 6.2.

4.7. Effect of distance to the rupture

In order to study the effects of distance to the source on inelastic displacement ratios mean inelastic displacement ratios in site class D were computed from earthquake ground motions in three groups having a different range of distances to the horizontal projection of the rupture (the so-called Joyner and Boore distance). A comparison of mean inelastic displacement ratios for different distances to the rupture for lateral strength ratios equal to 2 and 4 is shown



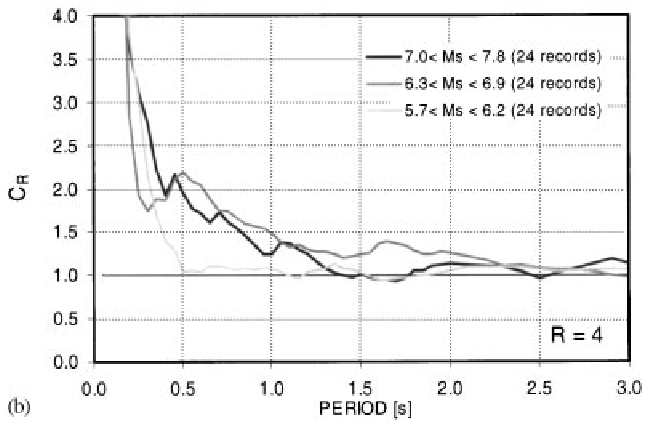


Figure 10. Effect of earthquake magnitude on mean inelastic displacement ratios for (a) $R=2$ and (b) $R=4$.

in Figure 11. It can be seen that, for this range of distances, changes in mean inelastic displacement ratios are relatively small. However, it should be noted that the ensemble of records considered here does not include near-fault records. In a recent study using only near-field records, Báez and Miranda [22] concluded that constant ductility inelastic displacement ratios for periods between 0.1 and 1.3 s for near-field records with forward directivity effects (i.e. those recorded in the horizontal component oriented perpendicular to the fault strike and where rupture moves towards the site) can be larger than those recorded farther away from the rupture or those not affected by forward directivity.

5. NON-LINEAR REGRESSION ANALYSES

For displacement-based design and, in general, in earthquake-resistant design it is desirable to have a simplified expression of inelastic displacement ratios to estimate maximum inelastic displacement demands from maximum elastic displacement demands for structures where the

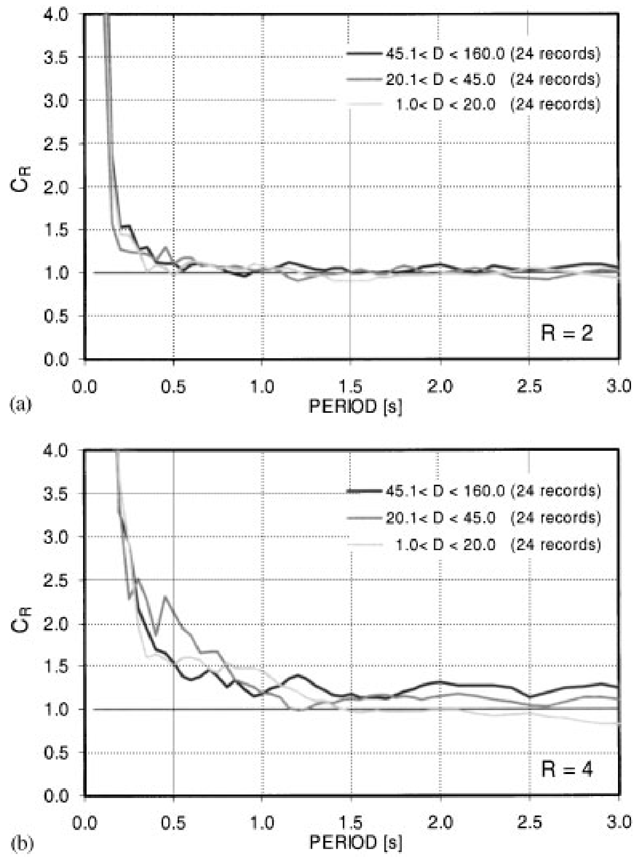


Figure 11. Effect of nearest distance to the horizontal projection of rupture on mean inelastic displacement ratios for (a) $R=2$ and (b) $R=4$.

lateral strength is known. In this investigation non-linear regression analyses were done in order to minimize the mean relative error given by

$$\bar{E} = \frac{1}{300n} \sum_{i=1}^{n} \sum_{j=1}^{50} \sum_{l=1}^{6} \frac{C_R \Delta_{\text{elastic},i,j} - \Delta_{\text{inelastic},i,j,k}}{\Delta_{\text{inelastic},i,j,k}} \tag{5}$$

where $\Delta_{\text{elastic},i,j}$ is the elastic displacement for the jth period when subjected to the ith record, $\Delta_{\text{elastic},i,j,k}$ is the inelastic displacement for the ith record, jth period and kth strength, $C_R\Delta_{\text{elastic},i,j}$ is the approximate inelastic displacement, and n is the number of records. The proposed equation is given by

$$C_R = 1 + \left[\frac{1}{a(T/T_s)^b} - \frac{1}{c}\right](R-1) \tag{6}$$

where R is the lateral strength ratio, T is the period of vibration of the system. T_s is the characteristic period at the site and a b, and c are constants that also depend on the site

Table IV. Site-dependent coefficients to be used in Equation (5).

Site class	a	b	c	T_s (s)
B	42	1.60	45	0.75
C	48	1.80	50	0.85
D	57	1.85	60	1.05

conditions. The Levenberg–Marquardt method [23] was used to compute the parameters a, b and c that minimize Equation (5). The resulting values of these parameters are given in Table IV. Equation (6) corresponds to a surface in the C_R–R–T space that provides estimates of mean inelastic displacement ratios as a function of R and T. This equation can be further simplified, with only slightly larger mean relative error, if coefficients $a = 50$, $b = 1.8$ and $c = 55$ are kept constant for all three site classes, which means that only T_s is changed from one class to another.

6. CONCLUSIONS

The purpose of this study was to assess inelastic displacement ratios that permit the estimation of maximum inelastic displacements from maximum elastic displacements for existing structures built on firm sites whose lateral strength is known. A statistical study has been presented of inelastic displacement ratios computed for SDOF systems with an elasto-plastic hysteretic behaviour with different levels of lateral strengths relative to the strength required to maintain the system elastic when subjected to a large number of earthquake ground motions. The following conclusions are drawn from the results of this investigation.

In the short-period spectral region, maximum inelastic displacement demands of systems with constant relative strength are on average much larger than maximum elastic demands. In this spectral region the ratio of maximum inelastic to maximum elastic displacement demand is strongly dependent on the period of vibration and on the lateral strength ratio. Constant relative strength inelastic displacement ratios increase as the lateral strength of the system decreases with respect to the lateral strength required to maintain the system elastic. For periods longer than 1.2 s and lateral strength ratios smaller than 6 maximum inelastic displacement demands are approximately equal to maximum elastic demands.

Coefficients of variation of inelastic displacement ratios increase with increasing lateral strength ratios. Dispersion is relatively large for lateral strength ratios higher than 4 and periods of vibration smaller than 1.5 s. With the exception of periods shorter than 0.15 s, coefficients of variation decrease with increasing period of vibration.

It was found that the effects of local site conditions on constant relative strength inelastic displacement ratios are slightly larger than those of constant ductility inelastic displacement ratios, however, in general the effect is still relatively small, particularly for periods longer than 1.2 s. Neglecting the effect of site conditions for structures with periods smaller than 1.5 s built on firm sites will typically result in errors less than 20% in the estimation of mean inelastic displacement ratios, whereas for periods longer than 1.5 s the errors are smaller than 10%. Differences are even smaller if the lateral strength ratio is equal to or smaller than 3.

Limiting periods dividing regions where the equal displacement rule is applicable from those where this rule is not applicable depend primarily on the lateral strength ratio, although they are also influenced by local site conditions. In general these limiting periods increase primarily as the lateral strength ratio increases and to a lesser degree as the average shear wave velocity in the upper 30 m of the site profile decreases.

For periods of vibration longer than 1.0 s changes in earthquake magnitude do not affect constant relative strength inelastic displacement ratios. Nevertheless, for systems with periods smaller than 1.0 s some dependence on magnitude was observed. In the short period region inelastic displacement ratios computed from ground motions recorded during earthquakes with magnitudes higher than 6.3 were found to be larger than those computed from records obtained in earthquakes with magnitudes between 5.7 and 6.2. Inelastic displacements are not affected by distance for sites located more than 10 km away from the horizontal projection of the rupture.

A simplified equation was derived using non-linear regression analyses to estimate inelastic displacement demands of existing structures with known lateral strength. Coefficients corresponding to different local site conditions were computed. The proposed equation minimizes mean relative errors between approximate and exact inelastic displacements.

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