Xu, K., and Igusa, T. (1992). "Dynamic characteristics of multiple substructures with closely spaced frequencies." *Earthq. Eng. and Str. Dyn.*, 21, 1059–1070.
Yamaguchi, H., and Harpornchar, N. (1993). "Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations." *Earthq. Eng. and Str. Dyn.*, 22, 51–62.
Yoneda, M., Fujino, Y., Kande, H., Yamamoto, A., Miyamoto, Y., Ando, O., Maeda, K., and Katayama, T. (1989). "A practical study of tuned liquid damper with application to the Sakitama Bridge." *J. Wind Engrg.*, 41, 105–106 (in Japanese).

Site-Dependent Strength-Reduction Factors

By Eduardo Miranda[1]

Abstract: Strength-reduction factors that are used to reduce linear elastic design spectra to account for the hysteretic energy dissipation of the structure are evaluated. The paper presents a summary of results of a statistical analysis of strength-reduction factors computed for single-degree-of-freedom systems undergoing different levels of inelastic deformation when subjected to a relatively large number of recorded earthquake ground motions. Special emphasis is given to the influence of soil conditions. Results indicate that for a given displacement ductility demand, the use of period-independent reduction factors is inadequate. Soil conditions can have an important effect on strength-reduction factors, particularly in the case of soft-soil sites. It is recommended that strength-reduction factors to be used in design be specified as a function of the period and inelastic capacity of the structure, and of at least two types of soil conditions—one for rock and relatively firm sites and another for soft-soil sites. Following these recommendations, simplified expressions to compute strength-reduction factors are proposed.

INTRODUCTION

Due to economic reasons, present design philosophy allows buildings and other types of structures to undergo inelastic deformations in the event of strong earthequake ground motions. As a result of this design philosophy, the design lateral strength prescribed in seismic codes is lower, and in some cases much lower, than the lateral strength required to maintain the structure in the elastic range.

Generally, the design lateral strength is prescribed by means of smoothed inelastic design response spectra (SIDRS). Although recent studies have concluded that a more rational design may be attained through SIDRS that are derived directly from statistical and probabilistic analyses of inelastic response spectra (Bertero et al. 1991; Miranda 1993), SIDRS currently used in design practice are the result of smoothed linear elastic response spectra (SLERS), which are then reduced to take into account the inelastic behavior in the structure.

Reductions in forces produced by the hysteretic energy dissipation capacity of the structure (i.e., reduction in forces due to nonlinear hysteretic behavior) are typically accounted for through the use of strength-reduction factors (sometimes also referred to as inelastic acceleration ratios) or through their reciprocals (typically referred to as deamplification factors). Thus, the assessment of reliable SIDRS derived from SLERS requires a good estimation of the strength-reduction factors.

Strength-reduction factors have been the topic of several investigations. One of the earliest and better known studies on strength-reduction factors is that of Newmark and Hall (1973) in which recommendations were made of reduction factors to be used in the short-, medium-, and long-period spectral regions. Riddell and Newmark (1979) proposed an improved set of reduction factors that was based on a statistical analysis of the response of

[1]Res. Engr., Dept. of Civ. Engrg., Swiss Fed. Inst. of Tech., CH-1015, Lausanne, Switzerland.

Note. Discussion open until May 1, 1994. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on December 18, 1992. This paper is part of the ***Journal of Structural Engineering***, Vol. 119, No. 12, December, 1993. ©ASCE, ISSN 0733-9445/93/0012-3503/$1.00 + $.15 per page. Paper No. 5305.

single-degree-of-freedom (SDOF) systems to 10 recorded earthquake ground motions. More recently, Riddell et al. (1989) presented approximate mean strength-reduction factor spectra computed as the ratio of mean elastic spectra to mean inelastic spectra. Nassar and Krawinkler (1991) studied mean reduction factors of bilinear and stiffness degrading systems when subjected to 15 ground motions recorded on firm sites in the western United States. They proposed approximate expressions to compute strength reduction factors as a function of ductility and period of vibration. With few exceptions, previous studies on reduction factors have not considered the influence of local site conditions. The reader is referred to Miranda (1991) for a detailed description of previous studies on inelastic response spectra and on strength-reduction factors.

The influence of soil conditions on reductions factors was first studied by Elghadamsi and Mohraz (1987), who considered ground motions recorded on rock sites and on alluvium sites. This study concluded that deamplification factors are not significantly influenced by soil conditions, and that for a given ductility and frequency one may deamplify the elastic response more for a structure on rock than for a structure on alluvium. Using a stochastic procedure, Peng et al. (1988) computed deamplification factors for rock and alluvium sites. Analogously to the earlier study, this investigation concluded that the effects of local soil conditions on inelastic spectra stem primarily from their effects on elastic response spectra; thus, soil conditions do not significantly influence strength-reduction factors. However, recent studies based on ground motion recorded during the 1989 Loma Prieta earthquake (Miranda and Bertero 1991; Krawinkler and Rahnama 1992) suggest that local site conditions may have a significant effect on strength-reduction factors, particularly in the case of soft soils.

The aim of this study is to improve the estimation of strength reductions in structures that behave inelastically during severe earthquake ground motions. The objectives of this paper are: (1) To study the main factors influencing strength-reduction factors; and (2) to provide approximate expressions that allow a rapid estimation of strength-reduction factors.

STRENGTH REDUCTION FACTORS

The equation of motion of a nonlinear SDOF system subjected to earthquake ground motions is given by

$$m\ddot{u}(t) + c\dot{u}(t) + F(t) = -m\ddot{u}_g(t) \quad \text{(1)}$$

where m, c, and $F(t)$ = mass, damping coefficient, and restoring force of the system, respectively; $u(t)$ = relative displacement; $u_g(t)$ = ground displacement; and overdot represents its derivative with respect to time. The initial period of the system is given by

$$T = 2\pi\left(\frac{m}{k}\right)^{1/2} = 2\pi\left(\frac{mu_y}{F_y}\right)^{1/2} \quad \text{(2)}$$

where k = initial stiffness of the system; F_y = system's yield strength; and u_y = yield displacement, respectively.

The level of inelastic deformation experienced by the system under a given ground motion is typically given by the displacement ductility ratio, which is defined as the ratio of maximum absolute relative displacement to its yield displacement

$$\mu = \frac{\max|u(t)|}{u_y} \quad \text{(3)}$$

An adequate design is produced when the structure is dimensioned and detailed in such a way that the local (story and member) ductility demands are smaller than their corresponding capacities. Thus, during the preliminary design of a structure there is a need to estimate the lateral strength (lateral load capacity) of the structure that is required in order to limit the global (structure) displacement ductility demand to a certain predetermined value, which results in the adequate control of local ductility demands.

The strength-reduction factor (i.e., reduction in strength demand due to nonlinear hysteretic behavior) R_μ is defined as the ratio of the elastic strength demand to the inelastic strength demand

$$R_\mu = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_i)} \quad \text{(4)}$$

where $F_y(\mu = 1)$ = lateral yielding strength required to maintain the system elastic; and $F_y(\mu = \mu_i)$ = lateral yielding strength required to maintain the displacement ductility demand μ less or equal to a predetermined target ductility ratio μ_i. Eq. (4) can be rewritten as

$$R_\mu = \frac{C_y(\mu = 1)}{C_y(\mu = \mu_i)} \quad \text{(5)}$$

where $C_y(\mu = 1)$ = seismic coefficient (yielding strength divided by the weight of the structure) required to avoid yielding; and $C_y(\mu = \mu_i)$ = minimum seismic coefficient required to control the displacement ductility demand to μ_i. As shown in Fig. 1, $C_y(\mu = 1)$ and $C_y(\mu = \mu_i)$ correspond to ordinates of a linear elastic response spectrum and a constant displacement ductility nonlinear response spectrum, respectively.

For design purposes, R_μ corresponds to the maximum reduction in strength that can be used in order to limit the displacement ductility demand to the

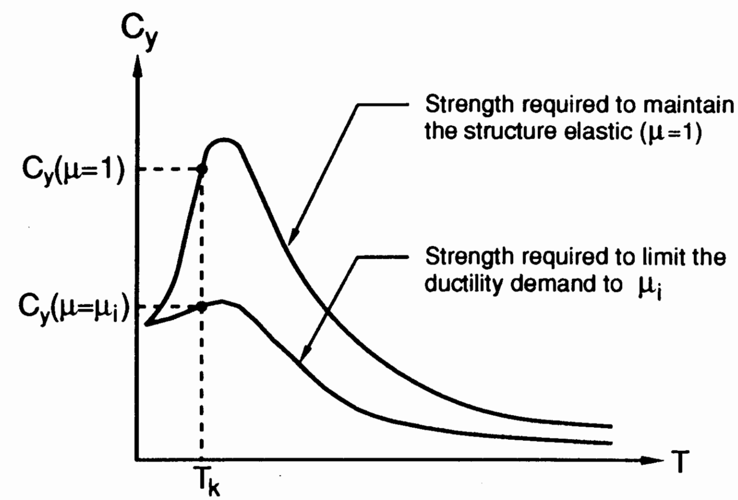

FIG. 1. Constant Displacement Ductility Nonlinear Response Spectra

predetermined ductility μ_i in a structure that will have a lateral strength equal to the design strength. An additional strength reduction can be considered in the design of a structure to account for the fact that structures usually have a lateral strength higher than the design strength. For a more detailed discussion on strength reductions due to overstrength the reader is referred to Osteraas et al. (1990), Miranda (1991), and Bertero et al. (1991).

Computation of $F_y(\mu = \mu_i)$ or $C_y(\mu = \mu_i)$ involves iteration (for each period and each target ductility) on the lateral strength F_y (or the seismic coefficient C_y) using (1) until the computed ductility demand under a given ground motion is, within a certain tolerance, the same as the target ductility.

Iteration on the lateral strength using (1) in some cases does not yield a unique result, that is, there can be more than one lateral strength that produces the same displacement ductility demand. In such cases, only the largest lateral strength is of interest for design purposes. This lateral strength capacity corresponds to the maximum strength reduction factor R_μ and the minimum strength required by the structure to limit the ductility demand to the target ductility.

STATISTICAL STUDY OF FORCE REDUCTION FACTORS

Earthquake Ground Motions

There is a general consensus that one of the largest sources of uncertainty in the estimation of the response of inelastic structures during earthquakes is the prediction of the intensity and characteristics of future earthquake ground motions at a given site. In this study, an effort was made to consider a relatively large number of recorded ground motion to study the effects of the variability of the characteristics of recorded ground motions on strength-reduction factors.

To study the influence of local site conditions on strength reduction factors, a group of 124 ground motions recorded on a wide range of soil conditions during various earthquakes was considered. The ground motions used in this investigation were recorded during the earthquakes listed in Table 1. Most of the selected records represent so-called free-field conditions. Complete listing of the records can be found in Miranda (1993).

Based on the local site conditions at the recording station, ground motions

TABLE 1. Earthquakes Considered in This Investigation

Earthquake (1)	Date (2)	Magnitude (3)
Imperial Valley, Calif.	May 18, 1940	$6.3(M_L)$
Kern County, Calif.	July 21, 1952	$7.7(M_S)$
San Francisco, Calif.	March 22, 1957	$5.3(M_L)$
Parkfield, Calif.	June 27, 1966	$5.6(M_L)$
San Fernando, Calif.	February 9, 1971	$6.5(M_L)$
Romania	March 4, 1977	$7.1(M_S)$
Miyagi-Ken-Oki, Japan	June 12, 1978	$7.4(M_S)$
Imperial Valley, Calif.	October 15, 1979	$6.6(M_L)$
Central Chile, Chile	March 3, 1985	$7.8(M_S)$
Michoacan, Mexico	September 19, 1985	$8.1(M_S)$
San Salvador, El Salvador	October 10, 1986	$5.4(M_S)$
Whittier-Narrows, Calif.	October 1, 1987	$6.1(M_L)$
Loma Prieta, Calif.	October 17, 1989	$7.1(M_S)$

were classified into three groups using a simple criterion similar to that used in present building codes. These three groups are: ground motions recorded on rock (38 records); ground motions recorded on alluvium (62 records); and ground motions recorded on very soft soil deposits characterized by low shear wave velocities (24 records). Records included in the latter category could be considered as representative of the soil type S_4 according to the soil classification of the *Uniform Building Code* (*Uniform* 1988).

Method of Analysis

For each earthquake record inelastic strength demands were computed for a family of 50 SDOF systems undergoing different levels of inelastic deformation. For a given period of vibration and a given target displacement ductility ratio, the inelastic strength demand $F_y(\mu = \mu_i)$ was computed by iteration on the system's lateral yielding strength until the displacement ductility demand computed with (1) and (3) was within 1% of the target ductility. The following target ductilities were selected: one (linear elastic behavior), two, three, four, five, and six. The number of iterations required to compute the maximum lateral strength that results in a ductility demand within 1% of the target ductility varies greatly depending on the period of vibration, the target ductility and the ground motion. In general, the number of iterations increases with increasing target ductility and decreasing period.

The SDOF systems considered in this study were characterized by bilinear hysteretic behavior with a postelastic stiffness equal to 3% of the elastic stiffness and a constant damping coefficient corresponding to a damping ratio ξ of 5% based on elastic properties and given by

$$c = 2m\xi\omega_e = 2\xi\sqrt{km} \qquad (6)$$

where ω_e = undamped elastic angular frequency on the system. On each iteration, response-time histories were computed by numerical step-by-step integration of (1) using the linear accleration method with a variable time step to minimize energy violations when changes in stiffness occur in the system.

After computing elastic and inelastic strength demands, strength-reduction factors were computed using (4). An R_μ spectrum can be constructed by plotting the strength-reduction factors of a family of SDOF systems undergoing a certain level of inelastic deformation under a given ground motion. An example of this kind of spectrum corresponding to a ground motion recorded near the epicenter of the 1989 Loma Prieta, California, earthquake is shown in Fig. 2.

Mean Strength Reduction Factors

Using the procedure just described, a total of 31,000 strength-reduction factors were computed (corresponding to 50 SDOF systems undergoing five different levels of inelastic deformation when subjected to 124 earthquake ground motions). Results were organized and analyzed statistically according to the period of vibration of the system, the target ductility and the soil condition where the ground motion was recorded.

For ground motions recorded on rock or alluvium sites, the strength-reduction factors were computed for a fixed set of periods between 0.05 s and 3.0 s. Mean strength-reduction factors computed for systems subjected to ground motions recorded on rock are shown in Fig. 3. As shown in this figure, the strength-reduction factors are characterized by the following features: first, the reduction factor increases with increasing target ductility,

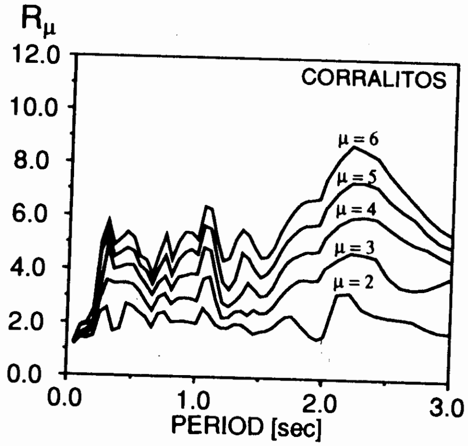

FIG. 2. Strength-Reduction Factors Computed for NS Component of Corralitos Record

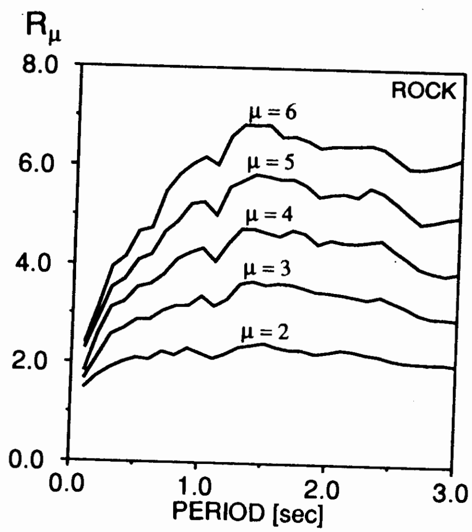

FIG. 3. Mean Strength-Reduction Factors for Systems Subjected to Ground Motions Recorded on Rock

with the rate of increase being period dependent; and second, for a given target ductility, the reduction factors exhibit an important variation with changes in period, particularly in the short-period region. In general, mean reduction factors in the long-period range are approximately constant and equal to the target ductility.

Mean strength-reduction factors computed for systems subjected to ground motions recorded on alluvium are shown in Fig. 4. As illustrated by this figure, strength-reduction factors for structures located on alluvium sites

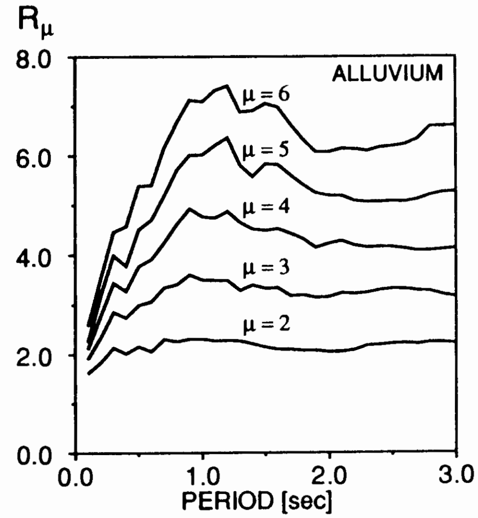

FIG. 4. Mean Strength-Reduction Factors for Systems Subjected to Ground Motions Recorded on Alluvium

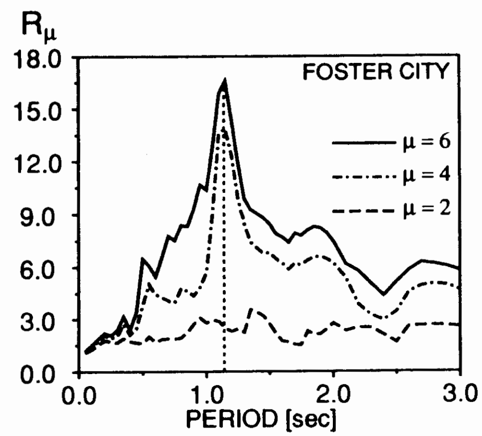

FIG. 5. Strength-Reduction Factors Computed for NS Component of Foster City Record

follow the same general trend of strength-reduction factors for structures on rock sites.

An example of a R_μ spectrum corresponding to a ground motion recorded during the 1989 Loma Prieta earthquake on a soft-soil site in the San Francisco Bay area is shown in Fig. 5. As shown in this figure, strength-reduction factors are very large around a period of 1.14 s. Typically, for very soft soil sites the period at which this peak is observed in the R_μ spectrum coincides with the predominant period of the ground motion (Miranda and Bertero 1991; Miranda 1991; Krawinkler and Rahnama 1992). Thus, the assessment of inelastic strength demands of structures located on soft-soil sites requires the estimation of the predominant period of the ground motion.

The predominant period of the ground motion T_g is defined by Miranda (1991) as the period at which the maximum input energy of a 5% damped linear elastic system is maximum throughout the whole period range. For a SDOF system, the maximum input energy is given by

$$E_I = \max\left[\int (m\ddot{u}_t)\, du_g\right] \quad \dots\dots (7)$$

where $\ddot{u}_t$ = total acceleration (ground plus relative acceleration) of the system. An example of the computation of the predominant period of the ground motion using this definition is shown in Fig. 6(*a*). The ground motion is the same record that was used to compute the R_μ spectrum shown in Fig. 5. It can be seen that the period at which the maximum strength-reduction factor is produced coincides with the period of maximum input energy.

If the linear elastic response spectrum of the ground motion is available, the predominant period of a ground motion recorded on a soft-soil site can also be estimated as the period at which the maximum relative velocity is produced (Miranda 1993). The maximum relative velocity is proportional to the "relative" kinetic energy. Thus, since absolute and relative kinetic energies are very close in the vicinity of the predominant period of the excitation (Uang and Bertero 1990), both procedures to estimate T_g will approximately yield the same result. The use of the second procedure to estimate T_g is exemplified in Fig. 6(*b*) for the Foster City ground motion. As demonstrated by this figure, both procedures produce approximately the same period.

Since the shape of a R_μ spectrum is strongly dependent on the value of T_g, obtaining the mean of R_μ versus T spectra of ground motion with significantly different predominant periods may result in a poor description of strength-reduction factors due to inelastic behavior for structures on soft-soil sites. Therefore, for ground motions in this soil category, strength-reductions factors were not computed for a fixed set of periods, but for a fixed set of T/T_g ratios.

Mean R_μ versus T/T_g spectra are shown in Fig. 7. As shown in this figure,

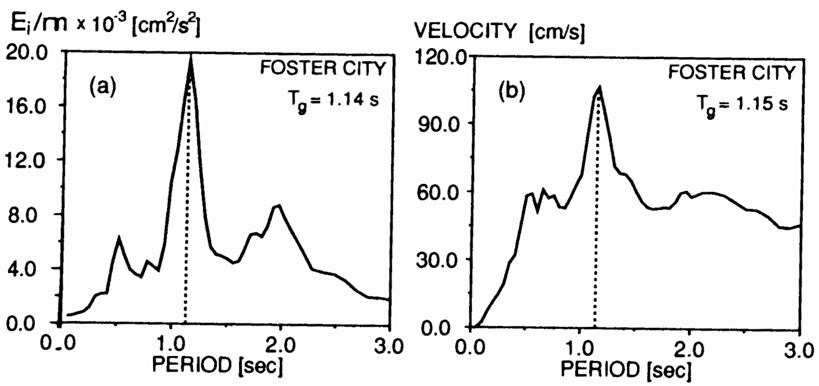

FIG. 6. Estimation of Predominant Period of Ground Motion: (*a*) Using Maximum Input Energy; and (*b*) Using Maximum Relative Velocity

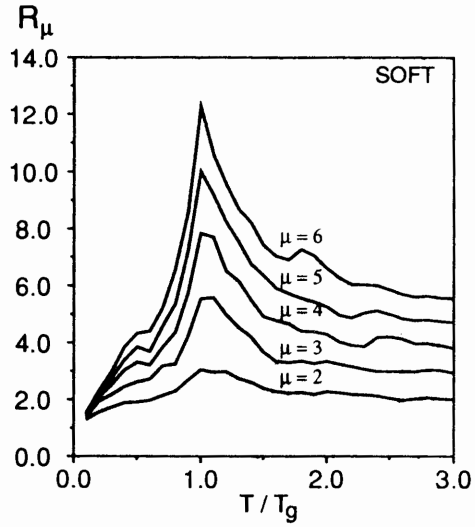

FIG. 7. Mean Strength-Reduction Factors for Systems Subjected to Ground Motions Recorded on Soft Soil

strength-reduction factors for ground motions recorded on soft-soil sites exhibit strong variations with changes in the T/T_g ratio. It can be seen that strength-reduction factors for structures built on soft-soil deposits are characterized by being much larger than the target ductility for periods near the predominant period of the ground motion (i.e., for $T \approx T_g$). For systems with periods shorter than two thirds of the predominant period of the ground motion, the strength-reduction factor due to inelastic behavior is smaller than the target ductility, whereas for systems with periods longer than one-and-a-half times the predominant period, the strength-reduction factor is approximately equal to the target ductility.

Variability of Strength Reduction Factors

The response of a nonlinear system subjected to earthquake ground motions is more sensitive to the characteristics of individual acceleration pulses and their sequence within a recorded acceleration time history than it is the response of a linear system. Therefore, for a given target ductility, the strength-reduction factor can exhibit great variations from one ground motion to another, even if both ground motions are similar (i.e., they have approximately the same intensity, duration, and frequency content). For the design of a structure this means that the lateral strength capacity required to avoid displacement ductility demands larger than a given limit can have important variations from one ground motion to another.

As mentioned before, strength-reduction factors increase with increasing ductility demands. For a given system with period of vibration T and a given target displacement ductility ratio, the strength-reduction factor will typically vary within a certain range when subjected to a family of ground motions. Thus, it is important to study not only the influence of the displacement ductility ratio on mean strength-reduction factors but also on the dispersion of these reduction factors. One way of evaluating the dispersion of strength-reduction factors is by computing the coefficient of variation (COV), which is defined as the ratio of the standard deviation to the mean.

Coefficients of variation of strength-reduction factors for systems subjected to ground motions recorded on rock and on alluvium are shown in Fig. 8. The coefficient of variation is shown for three displacement ductility ratios. As illustrated by this figure, with the exception of systems with very short periods ($T < 0.2$ s), coefficients of variation of strength-reduction factors exhibit only small variations with changes in the period of vibration. Regardless of the soil condition at the recording station, the dispersion in strength-reduction factors increases with increasing displacement ductility ratio.

Some of the factors that influence the intensity, frequency content, and duration of the ground motion at a given site are: the earthquake magnitude, the distance to the source, and the local site conditions. Thus, it is of great importance to study the influence of these factors on mean strength-reduction factors.

The influence of soil conditions on strength-reductions factors can be seen in Fig. 9 where mean R_μ spectra are plotted for systems undergoing displacement ductility demands of three and five when subjected to ground motions recorded on rock, on alluvium, and on soft-soil sites. For soft-soil sites, the mean R_μ spectra are plotted assuming a predominant period of the ground motion of 1.5 s. As shown in this figure, strength-reduction factors corresponding to ground motions recorded on alluvium are larger than those corresponding to ground motions recorded on rock for periods smaller than 1.2 s. Thus, in this period range one can design a structure on alluvium with a slightly smaller lateral strength capacity than that required to avoid the same level of inelastic deformation on a similar structure on a rock site. For systems with periods between 1.3 s and 2.4 s, the strength-reduction factors corresponding to ground motions recorded on rock are larger than those corresponding to ground motions recorded on alluvium.

Although difference exists between strength-reduction factors for rock sites and those of alluvium sites, these differences are relatively moderate when compared to the differences that exist between strength-reduction factors for soft-soil sites and strength-reduction factors for either rock or alluvium sites. As shown in the same figure, for systems on soft-soil sites

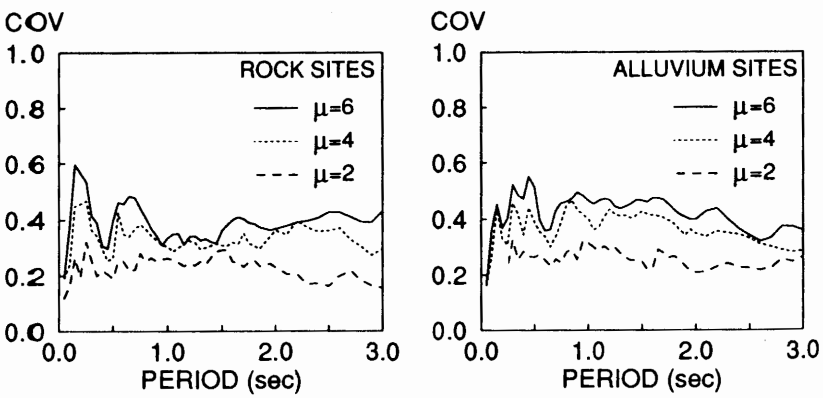

FIG. 8. Influence of Level of Inelastic Deformation on Dispersion of Strength-Reduction Factors

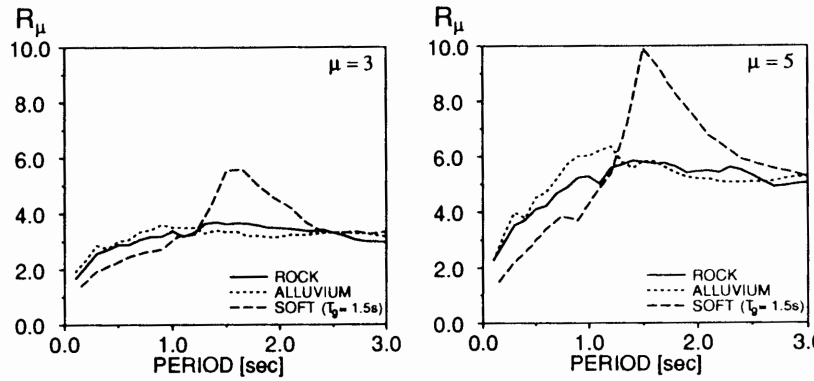

FIG. 9. Influence of Local Site Conditions on Strength-Reduction Factors

with periods between 1.3 s ($T \approx 0.85T_g$) and 2.3 s ($T \approx 1.5T_g$), the strength-reduction factor is much larger than those corresponding to systems with the same periods but located on either rock or alluvium sites.

In the short-period range, strength-reduction factors corresponding to systems on soft-soil sites are considerably smaller than those corresponding to systems on rock sites or to those corresponding to systems on alluvium sites. This observation has very important design implications. Mainly, that the use of strength-reduction factors derived from studies of systems subjected to ground motions recorded on rock and alluvium sites can lead to unconservative designs if used in the design of short-period structures located on soft-soil sites. For example, if displacement ductility demands larger than three want to be avoided on a structure with a period of 0.6 s, the use of mean strength-reduction factors derived with the use of ground motions recorded on rock or alluvium sites would result in a lateral strength capacity that is approximately one third of the lateral strength capacity that it is required to maintain the structure elastic (i.e., $R_\mu = 3$). However, if this strength-reduction factor is employed in the design of a structure located on soft soil, the mean displacement ductility demand would be approximately five, that is, 65% higher than the target ductility.

The influence of local site conditions on the dispersion of strength-reduction factors is shown in Fig. 10, where coefficients of variation of strength-reduction factors are plotted for systems undergoing displacement ductility ratios of three and five when subjected to ground motion recorded on rock, on alluvium, and on soft-soil sites. Periods of vibration for soft-soil sites correspond to an assumed predominant period of 1.2 s. It can be seen that, for a given displacement ductility ratio, the dispersion on the reduction factor is approximately the same for all three conditions. Thus, even though different soil conditions lead to different strength-reduction factors, their variability remains practically the same.

Earthquake magnitude and epicentral distance have been shown to influence elastic strength demands on SDOF systems (Silva and Green 1989). In the present investigation, the influence of earthquake magnitude on strength-reduction factors was studied by computing, for each soil condition, the mean R_μ spectra for ground motions recorded on earthquakes with three levels of magnitude. The influence of earthquake magnitude on mean reduction factors for systems undergoing displacement ductilities of two and

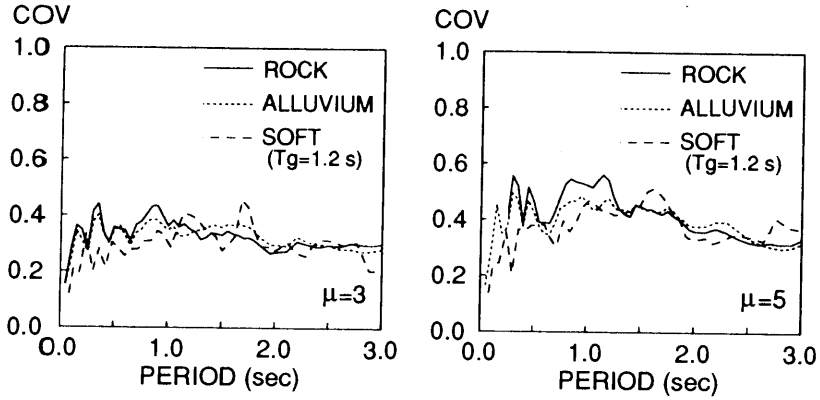

FIG. 10. Influence of Local Site Conditions on Dispersion of Strength-Reduction Factors

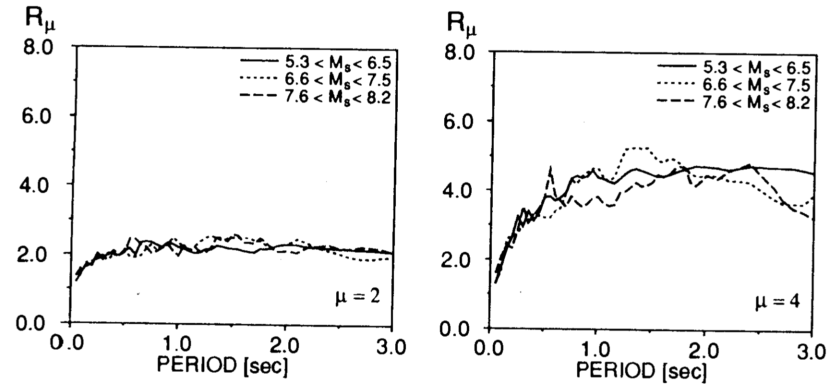

FIG. 11. Influence of Earthquake Magnitude of Strength-Reduction Factors for Systems Subjected to Ground Motions Recorded on Rock

four when subjected to ground motions recorded on rock during earthquakes with magnitude ranging from 5.3 to 8.1 are shown in Fig. 11. It can be seen that regardless of the level of ductility, the influence of magnitude on strength-reduction factors is negligible. Thus, the small effects of magnitude on inelastic strength demands stems primarily from its effects on elastic strength demand.

The influence of epicentral distance D on strength-reduction factors was studied by computing mean R_μ spectra for ground motions recorded within three groups of epicentral distances, approximately representing short, intermediate, and long epicentral distances. Mean R_μ spectra for systems undergoing displacement ductilities of two and four when subjected to ground motions recorded on rock at different epicentral distances are shown in Fig. 12. It can be seen that mean strength-reduction factors are practically the same for all three groups of epicentral distances. Thus, epicentral distances have a negligible effect on strength-reduction factors. A similar conclusion was reached by Krawinkler and Nassar (1990), who studied the effect of

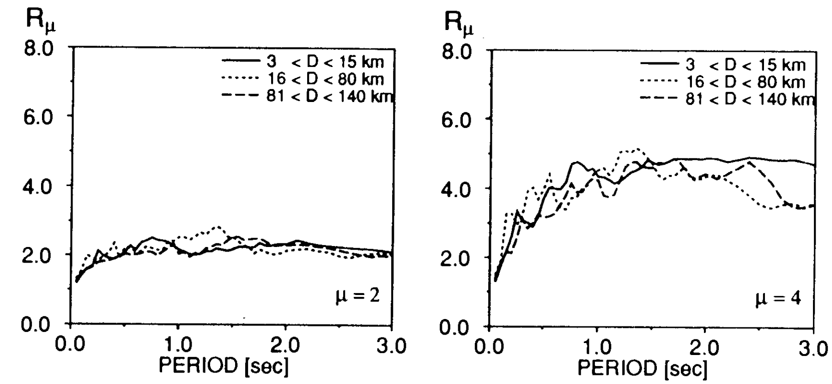

FIG. 12. Influence of Epicentral Distance on Strength-Reduction Factors for Systems Subjected to Ground Motions Recorded on Rock

epicentral distance on strength-reduction factors using 33 ground motions recorded during the 1987 Whittier-Narrows earthquake. In addition to the effect of epicentral distance, Krawinkler and Nassar studied the influence of stiffness degradation on strength-reduction factors. They concluded that stiffness degradation has a negligible effect on strength-reduction factors.

REGRESSION ANALYSES

For practical purposes, a simplified expression is desired to relate the strength-reduction factor due to hysteretic behavior R_μ to the displacement ductility ratio μ. Thus, for the design of a structure, the lateral strength capacity required to avoid displacement ductility demands larger than their corresponding capacities can be easily assessed for a given site-dependent SLERS. Similarly, if the lateral strength capacity is known, a simplified expression relating R_μ with μ permits a rapid estimation of the displacement ductility demand corresponding to a given site-dependent SLERS.

Some of the factors that influence R_μ are: displacement ductility ratio, period of vibration, local soil conditions, magnitude, epicentral distance, hysteretic behavior, and damping. Here only the first three factors, which are the ones that typically have a significant influence on R_μ, were considered while conducting regression analyses in order to obtain simplified expressions to compute strength-reduction factors. Therefore, the approximate force reduction factor $\hat{R}_\mu$ is given by

$$\hat{R}_\mu = f(\mu, T, SC) \quad (8)$$

where SC represents the soil conditions. Regardless of the soil condition, (8) has to satisfy the following conditions:

$$\lim_{T\to 0} \hat{R}_\mu = \lim_{T\to 0} f(\mu, T, SC) = 1 \quad (9)$$

$$\lim_{T\to \infty} \hat{R}_\mu = \lim_{T\to \infty} f(\mu\ T, SC) = \mu \quad (10)$$

$$\hat{R}_\mu = f(\mu, T, SC) = 1, \qquad \mu \le 1 \quad (11)$$

The form of the function described in (8) was chosen to be the following:

$$\hat{R}_\mu = \frac{\mu - 1}{\Phi} + 1 \geq 1 \qquad (12)$$

where Φ = function of μ, T, and the soil conditions at the site. Several forms of functions for Φ were considered, and regression analyses were conducted for each soil condition separately in order to fit the function Φ to the data obtained from nonlinear time-history analyses. For rock and alluvium sites the functions Φ that fit best mean strength-reduction factors are given by

$$\Phi = 1 + \frac{1}{10T - \mu T} - \frac{1}{2T}\exp\left[-\frac{3}{2}\left(\ln T - \frac{3}{5}\right)^2\right] \quad \text{(for rock sites)} \qquad (13)$$

$$\Phi = 1 + \frac{1}{12T - \mu T} - \frac{2}{5T}\exp\left[-2\left(\ln T - \frac{1}{5}\right)^2\right] \quad \text{(for alluvium sites)} \qquad (14)$$

A comparison between mean strength-reduction factors computed for systems subjected to ground motions recorded on rock sites and recorded on alluvium sites with those computed using (12)–(14) is shown in Fig. 13. It can be seen that the use of these simple equations leads to very good approximations of mean reduction factors due to inelastic behavior.

As shown in Fig. 7, mean strength-reduction factors for soft-soil conditions are characterized by important variations with changes in the T/T_g ratio. The assessment of this ratio depends on a good estimation of the fundamental period of vibration of the structure and of the predominant period of the ground motion, both of which are subjected to an important degree of uncertainty. Furthermore, the initial T/T_g ratio could also change during the earthquake as a result of nonstationarities on either the response of the soft-soil deposit or on the response of the structure. Thus, if the computed mean strength-reduction factors (Fig. 7) are directly used in design, even a small error in the estimation of the T/T_g ratio could lead to

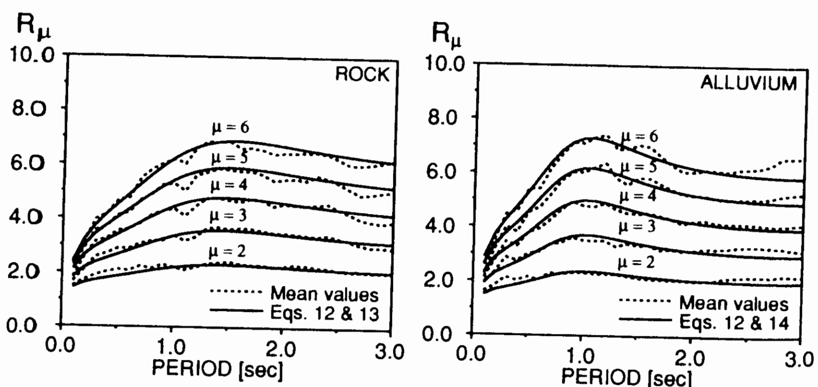

FIG. 13. Comparison of Mean Strength-Reduction Factors of Rock and Alluvium Sites with those Computed Using Eqs. (12)–(14)

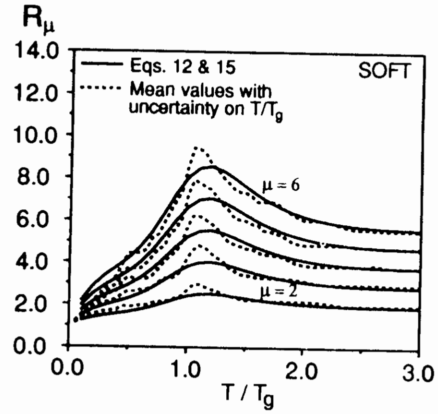

FIG. 14. Mean Strength-Reduction Factors of Soft Soil Sites Considering 10% Error in Estimation of T/T_g Ratio Compared to those Computed Using Eqs. (12) and (15)

significant errors in the estimation of R_μ, particularly for systems with fundamental periods of vibration close to the predominant period of the ground motion (i.e., $T/T_g \approx 1$).

Due to the important variations in R_μ with changes in the T/T_g ratio, combined with uncertainties in the estimation of the T/T_g ratio, it was decided to modify the computed strength-reduction factor spectra of ground motions recorded on soft-soil sites by considering a $\pm 10\%$ error in the estimation of the T/T_g ratio. For a given displacement ductility ratio and given T/T_g ratio, the modified strength-reduction factor was computed as the minimum strength-reduction factor in the spectral range limited by $0.9T/T_g$ and $1.1T/T_g$. Regression analyses were conducted to obtain a function Φ that, combined with (12), best fits the mean of modified strength-reduction factor spectra. This function Φ is given by

$$\Phi = 1 + \frac{T_g}{3T} - \frac{3T_g}{4T}\exp\left[-3\left(\ln\frac{T}{T_g} - \frac{1}{4}\right)^2\right] \quad \text{(for soft soil sites)} \qquad (15)$$

Strength-reduction factors computed using (12) and (15) and the mean of modified strength-reduction factors of systems subjected to ground motions recorded on soft-soil sites are compared in Fig. 14. As illustrated in this figure, the combined use of (12) and (15) provides, in general, good estimates of strength-reduction factors for structures located on soft-soil sites.

CONCLUSIONS

The primary purpose of this investigation was to assess the reduction in lateral strength demands produced by allowing nonlinear hysteretic behavior to take place in structures in the event of severe earthquake ground motions. For this purpose, a statistical study of strength-reduction factors was conducted. The statistical study comprised strength-reduction factors computed for SDOF systems undergoing different levels of inelastic deformation when subjected to a relatively large number of earthquake ground motions recorded on different local soil conditions. The following conclusions can be drawn from the results of this study.

The strength reduction factor, which controls displacement ductility demands, is primarily affected by the period of vibration of the system, the maximum tolerable inelastic displacement demand, and the soil conditions at the site.

For a given displacement ductility ratio, regardless of the soil conditions, strength-reduction factors exhibit important variations with changes in period, particularly in the short-period range where the use of a period-independent strength-reduction factor is clearly inadequate.

Periods at which strength-reduction factors become approximately equal to the displacement ductility ratio depend not only on the soil condition at the site but also on the level of inelastic deformation.

For systems on soft-soil sites, the assessment of the strength-reduction factor requires the estimation of the predominant period of the ground motion.

Strength-reduction factors of systems on alluvium sites are moderately different to those of systems on rock sites, whereas strength-reduction factors of systems on soft-soil sites are significantly different to those of systems on rock sites and to those of systems on alluvium.

Strength-reduction factors of systems on soft-soil sites with periods of vibration near the predominant period of the ground motion are typically much larger than the displacement ductility ratio.

For systems on soft-soil sites with periods smaller than two thirds of the predominant period, the strength-reduction factor is significantly smaller than that corresponding to systems with the same period on either rock or alluvium sites. Thus, the use of strength-reduction factors derived from studies of systems subjected to ground motions recorded on rock and alluvium sites can lead to unconservative designs if used in the design of short-period structures located on soft-soil sites.

The proposed expressions to compute site-dependent strength-reduction factors are relatively simple and provide a good estimation of mean strength-reduction factors derived from the statistical study presented herein.

ACKNOWLEDGMENTS

Part of the study presented in this paper was conducted while the writer was a graduate student at the University of California at Berkeley working under the supervision of Vitelmo V. Bertero, whose guidance is greatly appreciated.

APPENDIX I. REFERENCES

Bertero, V. V., Anderson, J. C., Krawinkler, H., and Miranda, E. (1991). "Design guidelines for ductility and drift limits." *Report no. EERC/UCB-91/15*, Earthquake Engrg. Res. Ctr., Univ. of California at Berkeley, Berkeley, Calif.

Elghadamsi, F. E., and Mohraz, B. (1987). "Inelastic earthquake spectra." *Earthquake Engrg. and Struct. Dynamics*, 15(2), 91–104.

Krawinkler, H., and Nassar, A. (1990). "Strength and ductility demands for SDOF and MDOF systems subjected to Whittier Narrows earthquake ground motions." *CSMIP-1990*, California Dept. of Conservation, Sacramento, Calif.

Krawinkler, H., and Rahnama, M. (1992). "Effects of soft soils on design spectra." *Proc., 10th World Conf. on Earthquake Engrg.*, Int. Assoc. for Earthquake Engrg., Madrid, Spain, 10, 5841–5846.

Miranda, E., and Bertero, V. V. (1991). "Evaluation of structural response factors using ground motions recorded during the Loma Prieta earthquake." *CSMIP-1991*, California Dept. of Conservation, Sacramento, Calif.

Miranda, E. (1991). "Seismic evaluation and upgrading of existing buildings," PhD thesis, University of California at Berkeley, Berkeley, Calif.

Miranda, E. (1993). "Evaluation of site-dependent inelastic seismic design spectra." *J. Struct. Engrg.*, ASCE, 119(5), 1319–1338.

Nassar, A., and Krawinkler, H. (1991). "Seismic demands for SDOF and MDOF systems." *Report no. 95*, The John A. Blume Earthquake Engrg. Ctr., Stanford Univ., Stanford, Calif.

Newmark, N. M., and Hall, W. J. (1973). "Procedures and criteria for earthquake resistant design." *Build. Res. Series No. 46*, Nat. Bureau of Standards, U.S. Dept. of Commerce, Washington, D.C., 209–236.

Osteraas, J. D., and Krawinkler, H. (1990). "Strength and ducility considerations in seismic design." *Report no. 90*, The John A Blume Earthquake Engrg. Ctr., Stanford Univ., Stanford, Calif.

Peng, M.-H., Elghadamsi, F. E., and Mohraz, B. (1988). "A stochastic procedure for nonlinear response spectra." *Proc., 9th World Conf. on Earthquake Engrg.*, Int. Assoc. for Earthquake Engrg., Tsukuba, Japan, V, 1069–1074.

Riddell, R., and Newmark, N. M. (1979). "Statistical analysis of the response of nonlinear systems subjected to earthquakes." *Structural Research Series No. 468*, Dept. of Civ. Engrg., Univ. of Illinois, Urbana, Ill.

Riddell, R., Hidalgo, P., and Cruz, E. (1989). "Response modification factors for earthquake resistant design of short period structures." *Earthquake Spectra*, 5(3), 571–590.

Silva, W. J., and Green, R. K. (1989). "Magnitude and distance scaling of response spectral shapes for rock sites with applications to North American tectonic environment." *Earthquake Spectra*, 5(3), 591–603.

Uniform building code. (1988). Int. Conf. of Build. Officials, Whittier, Calif.

APPENDIX II. NOTATION

The following symbols are used in this paper:

C_y = seismic coefficient;
c = damping coefficient;
D = epicentral distance;
E_I = maximum input energy;
F = restoring force;
F_y = yield resistance;
k = initial stiffness;
m = mass;
R_μ = strength-reduction factor;
$\hat{R}_\mu$ = approximate strength-reduction factor;
T = period of vibration;
T_g = predominant period of ground motion;
u = relative displacement;
$\ddot{u}_g$ = ground acceleration;
u_y = yield displacement;
μ = displacement ductility ratio;
ξ = damping ratio;
Φ = function necessary to compute approximate strength-reduction factors; and
ω_e = undamped elastic angular frequency.