Bavirisetty, R., Vinayagamoorthy, M., Duan, L. "Dynamic Analysis."
Bridge Engineering Handbook.
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

35
Dynamic Analysis

Rambabu Bavirisetty
California Department of Transportation

Murugesu Vinayagamoorthy
California Department of Transportation

Lian Duan
California Department of Transportation

35.1 Introduction

The primary purpose of this chapter is to present dynamic methods for analyzing bridge structures when subjected to earthquake loads. Basic concepts and assumptions used in typical dynamic analysis are presented first. Various approaches to bridge dynamics are then discussed. A few examples are presented to illustrate their practical applications.

35.1.1 Static vs. Dynamic Analysis

The main objectives of a structural analysis are to evaluate structural behavior under various loads and to provide the information necessary for design, such as forces, moments, and deformations. Structural analysis can be classified as *static* or *dynamic*: while *statics* deals with time-independent loading, *dynamics* considers any load where the magnitude, direction, and position vary with time. Typical dynamic loads for a bridge structure include vehicular motions and wave actions such as winds, stream flow, and earthquakes.

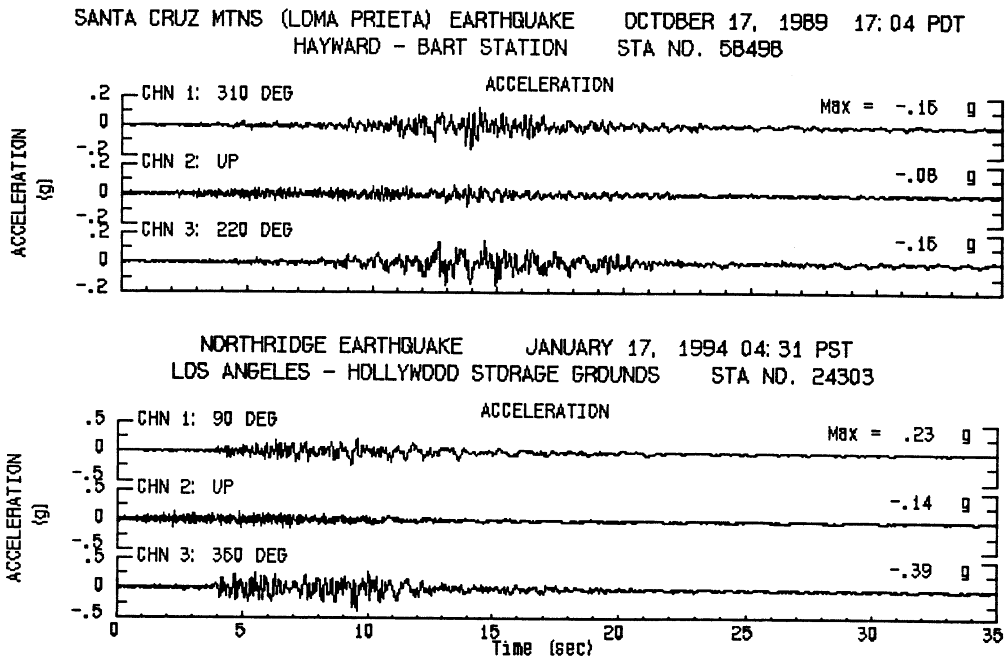

FIGURE 35.1 Ground motions recorded during recent earthquakes.

35.1.2 Characteristics of Earthquake Ground Motions

An earthquake is a natural ground movement caused by various phenomena including global tectonic processes, volcanism, landslides, rock-bursts, and explosions. The global tectonic processes are continually producing mountain ranges and ocean trenches at the Earth's surface and causing earthquakes. This section briefly discusses the earthquake input for seismic bridge analysis. Detailed discussions of ground motions are presented in Chapter 33.

Ground motion is represented by the time history or seismograph in terms of acceleration, velocity, and displacement for a specific location during an earthquake. Time history plots contain complete information about the earthquake motions in the three orthogonal directions (two horizontal and one vertical) at the strong-motion instrument location. Acceleration is usually recorded by strong-motion accelerograph and the velocities and displacements are determined by numerical integration. The accelerations recorded at locations that are approximately the same distance away from the epicenter may differ significantly in duration, frequency content, and amplitude due to different local soil conditions. Figure 35.1 shows several time histories of recent earthquakes.

From a structural engineering view, the most important characteristics of an earthquake are the peak ground acceleration (PGA), duration, and frequency content. The PGA is the maximum acceleration and represents the intensity of a ground motion. Although the ground velocity may be a more significant measure of intensity than the acceleration, it is not often measured directly, but determined using supplementary calculations [1]. The duration is the length of time between the first and the last peak exceeding a specified strong motion level. The longer the duration of a strong motion, the more energy is imparted to a structure. Since the elastic strain energy absorbed by a structure is very limited, a longer strong earthquake has a greater possibility to enforce a structure into the inelastic range. The frequency content can be represented by the number of zero crossings per second in the accelerogram. It is well understood that when the frequency of a regular disturbing force is the same as the natural vibration frequency of a structure (resonance), the oscillation of structure can be greatly magnified and effects of damping become minimal. Although earthquake motions are never as regular as a sinusoidal waveform, there is usually a period that dominates the response.

Since it is impossible to measure detailed ground motions for all structure sites, the rock motions or ground motions are estimated at a fault and then propagated to the Earth surface using a computer program considering the local soil conditions. Two guidelines [2, 3] recently developed by the California Department of Transportation provide the methods to develop seismic ground motions for bridges.

35.1.3 Dynamic Analysis Methods for Seismic Bridge Design

Depending on the seismic zone, geometry, and importance of the bridge, the following analysis methods may be used for seismic bridge design:

- The single-mode method (single-mode spectral and uniform load analysis) [4,5] assumes that seismic load can be considered as an equivalent static horizontal force applied to an individual frame in either the longitudinal or transverse direction. The equivalent static force is based on the natural period of a single degree of freedom (SDOF) and code-specified response spectra. Engineers should recognize that the single-mode method (sometimes referred to as equivalent static analysis) is best suited for structures with well-balanced spans with equally distributed stiffness.
- Multimode spectral analysis assumes that member forces, moments, and displacements due to seismic load can be estimated by combining the responses of individual modes using the methods such as complete quadratic combination (CQC) method and the square root of the sum of the squares (SRSS) method. The CQC method is adequate for most bridge systems [6], and the SRSS method is best suited for combining responses of well-separated modes.
- The multiple support response spectrum (MSRS) method provides response spectra and the peak displacements at individual support degrees of freedom by accurately accounting for the spatial variability of ground motions including the effects of incoherence, wave passage, and spatially varying site response. This method can be used for multiply supported long structures [7].
- The time history method is a numerical step-by-step integration of equations of motion. It is usually required for critical/important or geometrically complex bridges. Inelastic analysis provides a more realistic measure of structural behavior when compared with an elastic analysis.

Selection of the analysis method for a specific bridge structure should not be purely based on performing structural analysis, but be based on the effective design decisions [8]. Detailed discussions of the above methods are presented in the following sections.

35.2 Single-Degree-of-Freedom System

The familiar spring–mass system represents the simplest dynamic model and is shown in Figure 35.2a. When the *idealized, undamped* structures are excited by either moving the support or by displacing the mass in one direction, the mass oscillates about the equilibrium state forever without coming to rest. But, real structures do come to rest after a period of time due to a phenomenon called *damping*. To incorporate the effect of the damping, a massless viscous damper is always included in the dynamic model, as shown in Figure 35.2b.

In a dynamic analysis, the number of displacements required to define the displaced positions of all the masses relative to their original positions is called the number of degrees of freedom (DOF). When a structural system can be idealized with a single mass concentrated at one location and moved only in one direction, this dynamic system is called an SDOF system. Some structures,

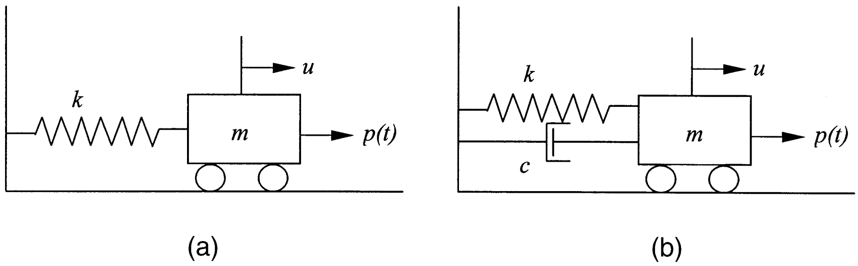

FIGURE 35.2 Idealized dynamic model. (a) Undamped SDOF system; (b) damped SDOF system.

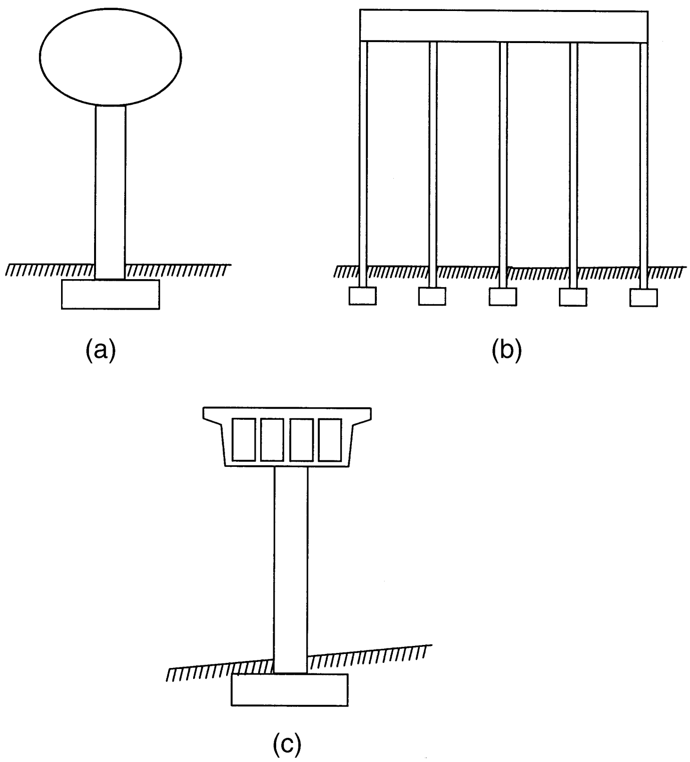

FIGURE 35.3 Examples of SDOF structures. (a) Water tank supported by single column; (b) one-story frame building; (c) two-span bridge supported by single column.

such as a water tank supported by a single-column, one-story frame structure and a two-span bridge supported by a single column, could be idealized as SDOF models (Figure 35.3).

In the SDOF system shown in Figure 35.3c, the mass of the bridge superstructure is the mass of the dynamic system. The stiffness of the dynamic system is the stiffness of the column against side sway and the viscous damper of the system is the internal energy absorption of the bridge structure.

35.2.1 Equation of Motion

The response of a structure depends on its mass, stiffness, damping, and applied load or displacement. The structure could be excited by applying an external force $p(t)$ on its mass or by a ground motion $u(t)$ at its supports. In this chapter, since the seismic loading is induced by exciting the support, we focus mainly on the equations of motion of an SDOF system subjected to ground excitation.

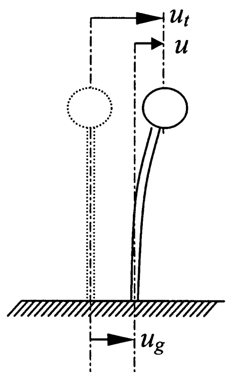

FIGURE 35.4 Earthquake–induced motion of an SDOF system.

The displacement of the ground motion u_g, the total displacement of the single mass u_t, and the relative displacement between the mass and ground u (Figure 35.4) are related by

$$u_t = u + u_g \tag{35.1}$$

By applying Newton's law and D'Alembert's principle of dynamic equilibrium, it can be shown that

$$f_I + f_D + f_S = 0 \tag{35.2}$$

where f_I is the inertial force of the single mass and is related to the acceleration of the mass by $f_I = m\ddot{u}_t$; f_D is the damping force on the mass and related to the velocity across the viscous damper by $f_D = c\dot{u}$; f_S is the elastic force exerted on the mass and related to the relative displacement between the mass and the ground by $f_S = ku$, where k is the spring constant; c is the damping ratio; and m is the mass of the dynamic system.

Substituting these expressions for f_I, f_D, and f_S into Eq. (35.2) gives

$$m\ddot{u}_t + c\dot{u} + ku = 0 \tag{35.3}$$

The equation of motion for an SDOF system subjected to a ground motion can then be obtained by substituting the Eq. (35.1) into Eq. (35.3), and is given by

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \tag{35.4}$$

35.2.2 Characteristics of Free Vibration

To determine the characteristics of the oscillations such as the time to complete one cycle of oscillation (T_n) and number of oscillation cycles per second (ω_n), we first look at the *free* vibration of a dynamic system. Free vibration is typically initiated by disturbing the structure from its equilibrium state by an external force or displacement. Once the system is disturbed, the system vibrates without any external input. Thus, the equation of motion for free vibration can be obtained by setting $\ddot{u}_g$ to zero in Eq. (35.4) and is given by

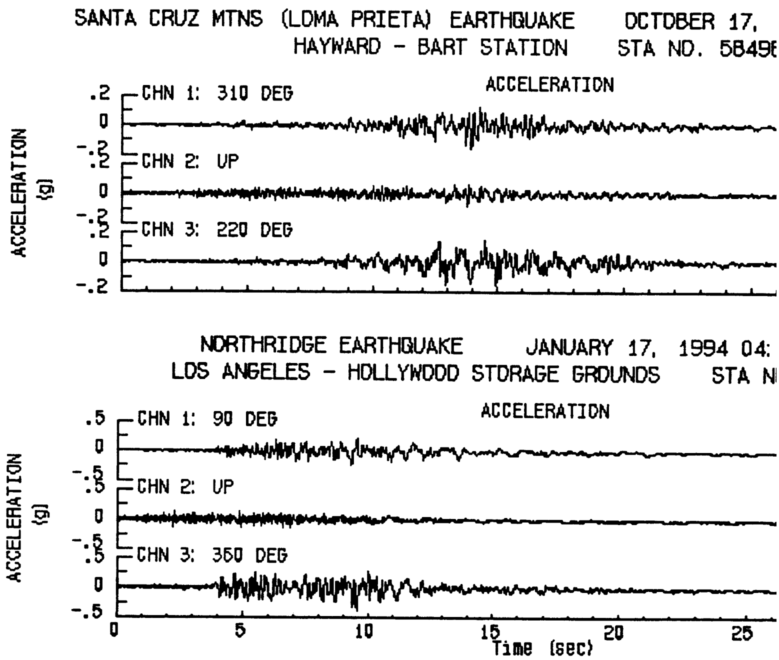

FIGURE 35.5 Typical response of an SDOF system. (a) Undamped; (b) damped.

$$m\ddot{u} + c\dot{u} + ku = 0 \qquad (35.5)$$

Dividing the Equation (35.5) by its mass m will result in

$$\ddot{u} + \left(\frac{c}{m}\right)\dot{u} + \left(\frac{k}{m}\right)u = 0 \qquad (35.6)$$

$$\ddot{u} + 2\xi\omega_n + \omega_n^2 u = 0 \qquad (35.7)$$

where $\omega_n = \sqrt{k/m}$ the natural circular frequency of vibration or the undamped frequency; $\xi = c/c_{cr}$ the damping ratio; $c_{cr} = 2m\omega_n = 2\sqrt{km} = 2k/\omega_n$ the critical damping coefficient.

Figure 35.5a shows the response of a typical idealized, *undamped* SDOF system. The time required for the SDOF system to complete one cycle of vibration is called the natural period of vibration (T_n) of the system and is given by

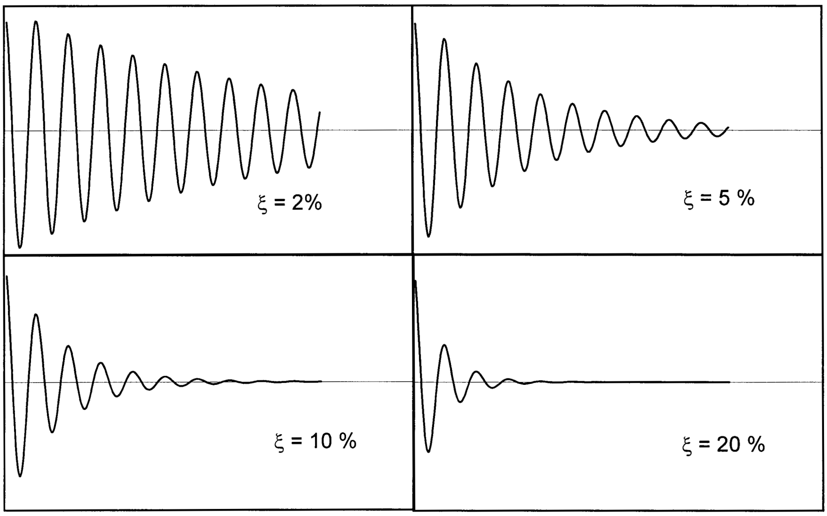

FIGURE 35.6 Response of an SDOF system for various damping ratios.

$$T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{k}} \tag{35.8}$$

Furthermore, the natural cyclic frequency of vibration f_n is given by

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{35.9}$$

Figure 35.5b shows the response of a typical *damped* SDOF structure. The circular frequency of the vibration or damped vibration frequency of the SDOF structure, ω_d, is given by $\omega_d = \omega_n\sqrt{1-\xi^2}$.

The damped period of vibration (T_d) of the system is given by

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\xi^2}}\sqrt{\frac{m}{k}} \tag{35.10}$$

When $\xi = 1$ or $c = c_{cr}$ the structure returns to its equilibrium position without oscillating and is referred to as a critically damped structure. When $\xi > 1$ or $c > c_{cr}$, the structure is *overdamped* and comes to rest without oscillating, but at a slower rate. When $\xi < 1$ or $c < c_{cr}$, the structure is *underdamped* and oscillates about its equilibrium state with progressively decreasing amplitude. Figure 35.6 shows the response of SDOF structures with different damping ratios.

For structures such as buildings, bridges, dams, and offshore structures, the damping ratio is less than 0.15 and thus can be categorized as *underdamped* structures. The basic dynamic properties estimated using damped or undamped assumptions are approximately the same. For example, when $\xi = 0.10$, $\omega_d = 0.995\omega_n$, and $T_d = 1.01T_n$.

Damping dissipates the energy out of a structure in opening and closing of microcracks in concrete, stressing of nonstructural elements, and friction at the connection of steel members. Thus, the damping coefficient accounts for all energy-dissipating mechanisms of the structure and can only be estimated by experimental methods. Two seemingly identical structures may have slightly different material properties and may dissipate energy at different rates. Since damping does not play an important quantitative role except for resonant responses in structural responses, it is common to use average damping ratios based on the types of construction materials. Relative damping ratios for common types of structures, such as welded metal of 2 to 4%, bolted metal structures of 4 to 7%, prestressed concrete structures of 2 to 5%, reinforced-concrete structures of 4 to 7% and wooden structures of 5 to 10%, are recommended by Chmielewski et al. [9].

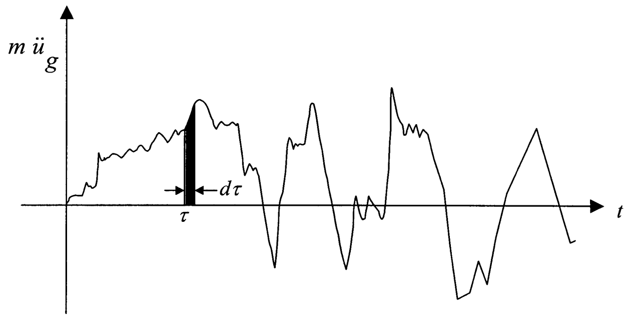

FIGURE 35.7 Induced earthquake force vs. time on an SDOF system.

35.2.3 Response to Earthquake Ground Motion

A typical excitation of an earth movement is shown in Figure 35.7. The basic equation of motion of an SDOF system is expressed in Eq. (35.4). Since the excitation force $m\ddot{u}_g$ cannot be described by simple mathematical expression, closed-form solutions for Eq. (35.4) are not available. Thus, the entire ground excitation needs to be treated as a superposition of short-duration impulses to evaluate the response of the structure to the ground excitation. An impulse is defined as the product of the force times duration. For example, the impulse of the force at time τ during the time interval $d\tau$ equals $-m\ddot{u}_g(\tau)d\tau$ and is represented by the shaded area in Figure 35.7. The total response of the structure for the earthquake motion can then be obtained by integrating all responses of the increment impulses. This approach is sometimes referred to as "time history analysis." Various solution techniques are available in the technical literature on structural dynamics [1,10].

In seismic structural design, designers are interested in the maximum or extreme values of the response of a structure as discussed in the following sections. Once the dynamic characteristics (T_n and ω_n) of the structure are determined, the maximum displacement, moment, and shear on the SDOF system can easily be estimated using basic principles of mechanics.

35.2.4 Response Spectra

The response spectrum is a relationship of the peak values of a response quantity (acceleration, velocity, or displacement) with a structural dynamic characteristic (natural period or frequency). Its core concept in earthquake engineering provides a much more convenient and meaningful measure of earthquake effects than any other quantity. It represents the peak response of all possible SDOF systems to a particular ground motion.

Elastic Response Spectrum

This, the response spectrum of an elastic structural system, can be obtained by the following steps [10]:

1. Define the ground acceleration time history (typically at a 0.02-second interval).
2. Select the natural period T_n and damping ratio ξ of an elastic SDOF system.
3. Compute the deformation response $u(t)$ using any numerical method.
4. Determine u_o, the peak value of $u(t)$.
5. Calculate the spectral ordinates by $D = u_o$, $V = 2\pi D / T_n$, and $A = (2\pi / T_n)^2 D$.
6. Repeat Steps 2 and 5 for a range of T_n and ξ values for all possible cases.
7. Construct results graphically to produce three separate spectra as shown in Figure 35.8 or a combined tripartite plot as shown in Figure 35.9.

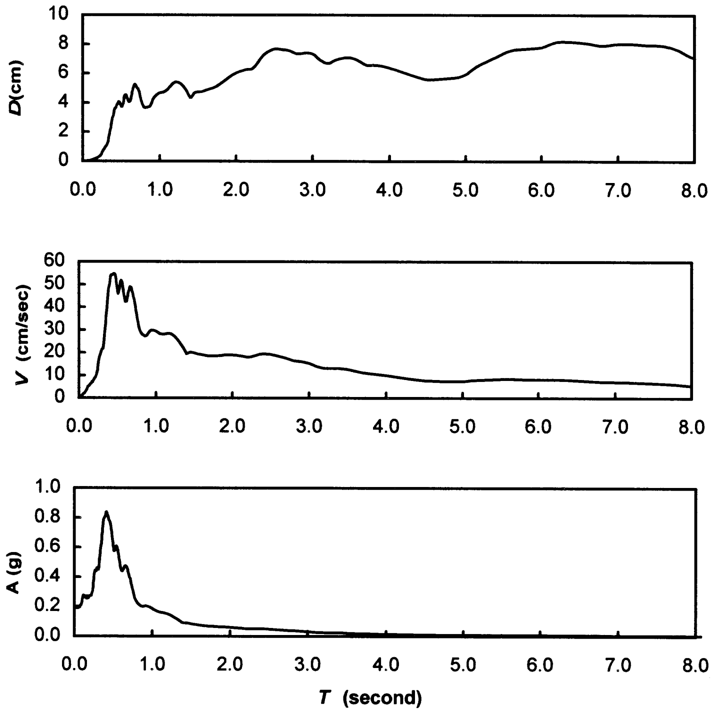

FIGURE 35.8 Example of response spectra (5% critical damping) for Loma Prieta 1989 motion.

It is noted that although three spectra (displacement, velocity, and acceleration) for a specific ground motion contain the same information, each provides a physically meaningful quantity. The displacement spectrum presents the peak displacement. The velocity spectrum is related directly to the peak strain energy stored in the system. The acceleration spectrum is related directly to the peak value of the equivalent static force and base shear.

A response spectrum (Figure 35.9) can be divided into three ranges of periods [10]:

- Acceleration-sensitive region (very short period region): A structure with a very short period is extremely stiff and expected to deform very little. Its mass moves rigidly with the ground and its peak acceleration approximately equals the ground acceleration.
- Velocity-sensitive region (intermediate-period region): A structure with an intermediate period responds greatly to the ground velocity than other ground motion parameters.
- Displacement-sensitive region (very long period region): A structure with a very long period is extremely flexible and expected to remain stationary while the ground moves. Its peak deformation is closer to the ground displacement. The structural response is most directly related to ground displacement.

Elastic Design Spectrum

Since seismic bridge design is intended to resist future earthquakes, use of a response spectrum obtained from a particular past earthquake motion is inappropriate. In addition, jagged spectrum values over small ranges would require an unreasonable accuracy in the determination of the structure period [11]. It is also impossible to predict a jagged response spectrum in all its details for a ground motion that may occur in the future. To overcome these shortcomings, the elastic design spectrum, a smoothened idealized response spectrum, is usually developed to represent the envelopes of ground motions recorded at the site during past earthquakes. The development of an elastic design spectrum is based on statistical analysis of the response spectra for the ensemble of ground motions. Figure 35.10 shows a set of elastic design spectra in Caltrans Bridge Design Specifications [12]. Figure 35.11 shows project-specific acceleration response spectra for the California Sonoma Creek Bridge.

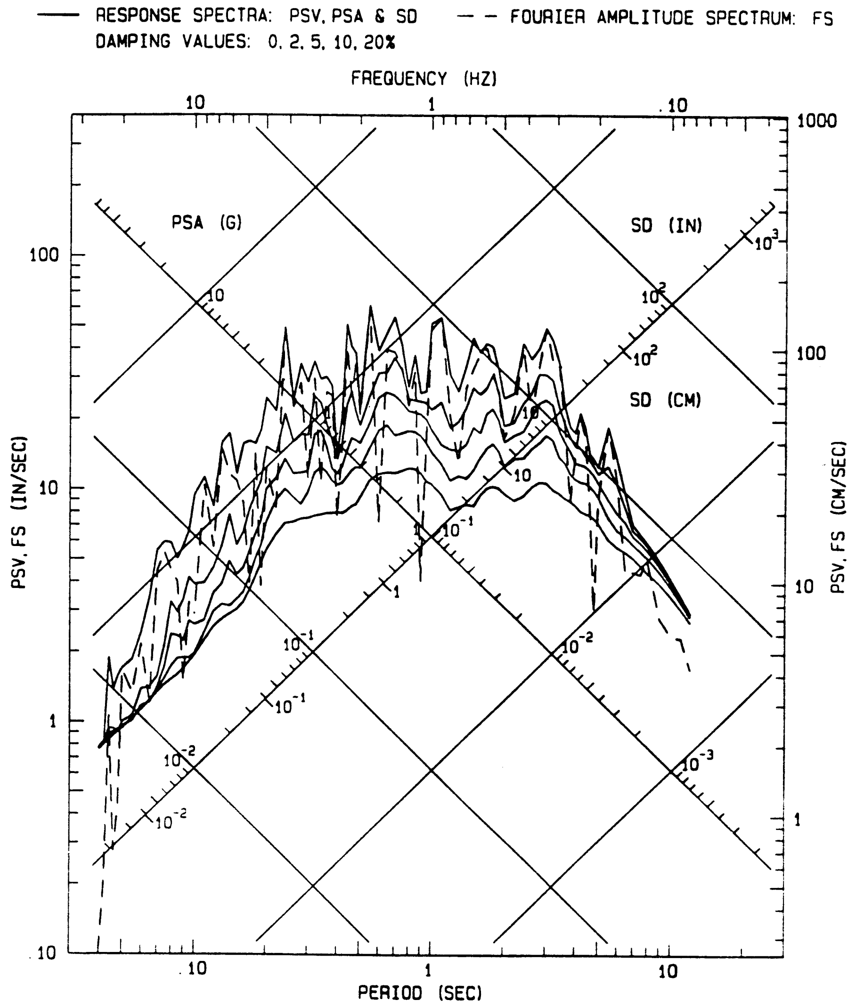

FIGURE 35.9 Tripartite plot–response spectra (1994 Northridge Earthquake, Arleta–Rordhoff Ave. Fire Station).

Engineers should recognize the conceptual differences between a response spectrum and a design spectrum [10]. A response spectrum is only the peak response of all possible SDOF systems due to a particular ground motion, whereas a design spectrum is a specified level of seismic design forces or deformations and is the envelope of two different elastic design spectra. The elastic design spectrum provides a basis for determining the design force and deformation for elastic SDOF systems.

Inelastic Response Spectrum

A bridge structure may experience inelastic behavior during a major earthquake. The typical elastic and elastic–plastic responses of an idealized SDOF to severe earthquake motions are shown in Figure 35.12. The input seismic energy received by a bridge structure is dissipated by both viscous damping and yielding (localized inelastic deformation converting into heat and other irrecoverable forms of energy). Both viscous damping and yielding reduce the response of inelastic structures compared with elastic structures. Viscous damping represents the internal friction loss of a structure when deformed and is approximately a constant because it depends mainly on structural materials. Yielding, on the other hand, varies depending on structural materials, structural configurations, and loading patterns and histories. Damping has negligible effects on the response of structures for

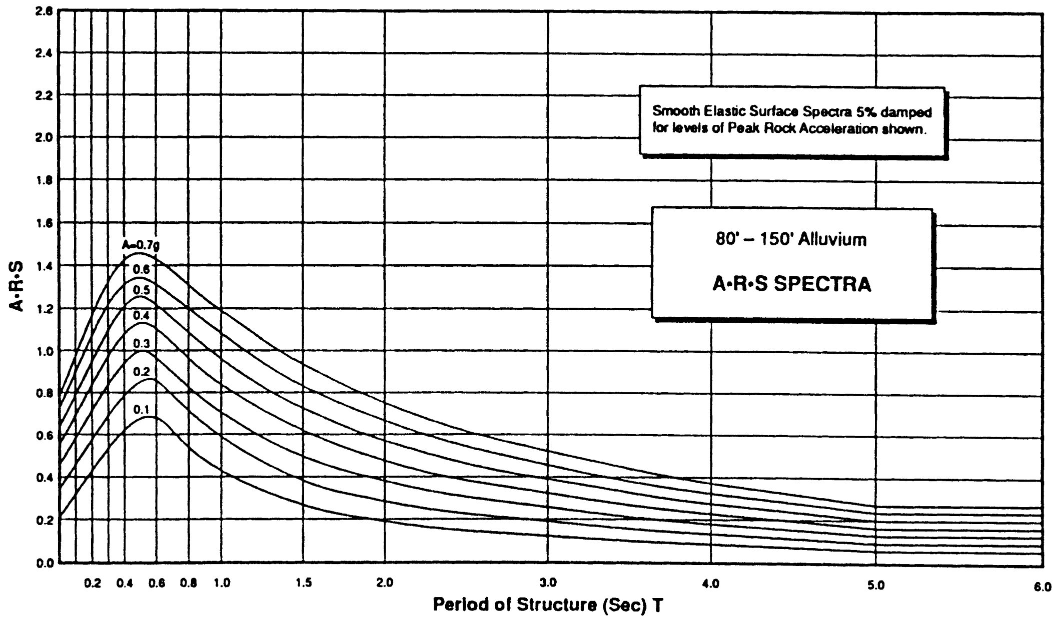

FIGURE 35.10 Typical Caltrans elastic design response spectra.

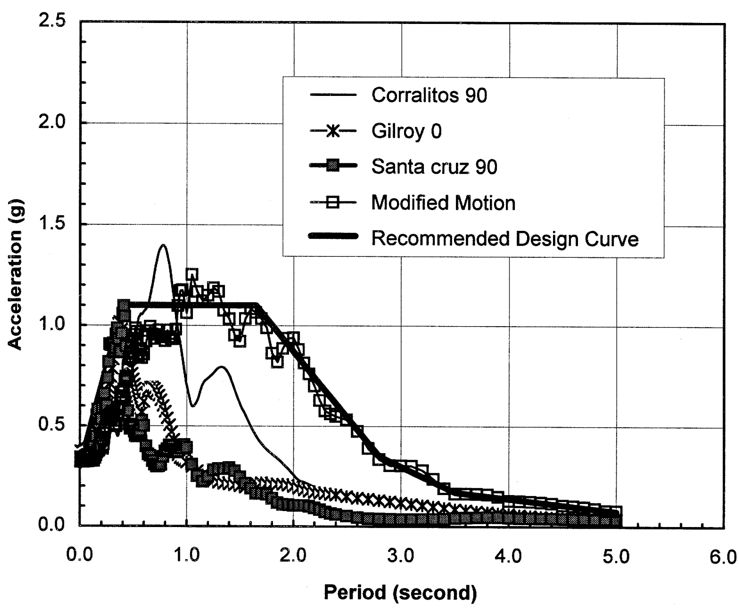

FIGURE 35.11 Acceleration response spectra for Sonoma Creek Bridge.

the long-period and short-period systems and is most effective in reducing response of structures for intermediate-period systems.

In seismic bridge design, a main objective is to ensure that a structure is capable of deforming in a ductile manner when subjected to a larger earthquake loading. It is desirable to consider the inelastic response of a bridge system to a major earthquake. Although a nonlinear inelastic dynamic analysis is not difficult in concept, it requires careful structural modeling and intensive computing

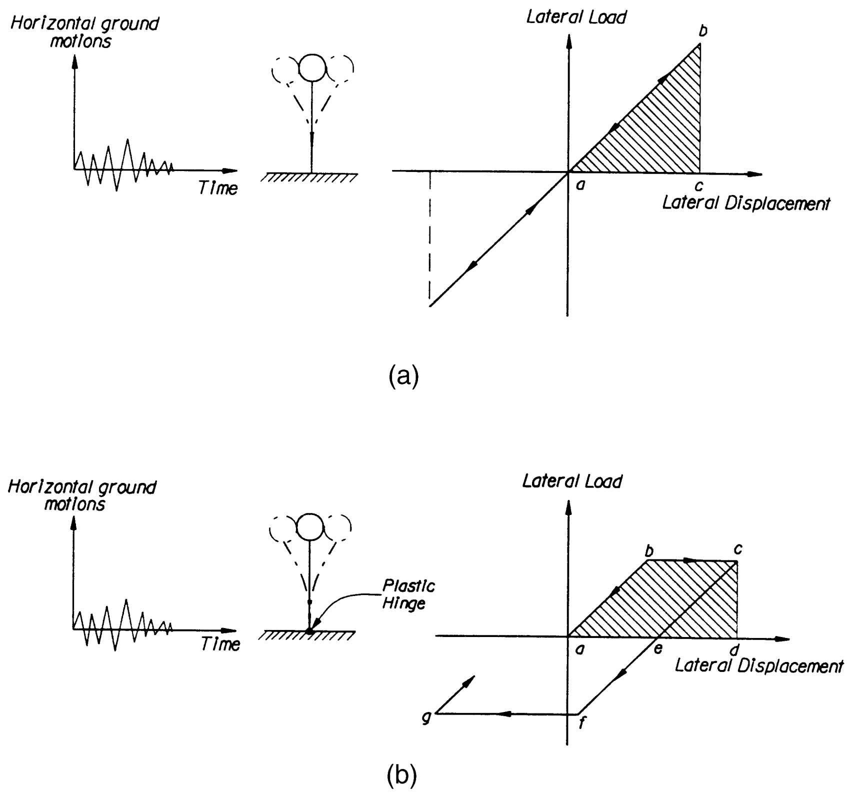

FIGURE 35.12 Response of an SDOF to earthquake ground motions. (a) Elastic system; (b) inelastic system.

effort [8]. To consider inelastic seismic behavior of a structure without performing a true nonlinear inelastic analysis, the ductility-factor method can be used to obtain the inelastic response spectra from the elastic response spectra. The ductility of a structure is usually referred as the displacement ductility factor μ defined by (Figure 35.13):

$$\mu = \frac{\Delta_u}{\Delta_y} \tag{35.11}$$

where Δ_u is ultimate displacement capacity and Δ_y is yield displacement.

The simplest approach to developing the inelastic design spectrum is to scale the elastic design spectrum down by some function of the available ductility of a structural system:

$$ARS_{\text{inelastic}} = \frac{ARS_{\text{elastic}}}{f(\mu)} \tag{35.12}$$

$$f(\mu) = \begin{cases} 1 & \text{for } T_n \le 0.03 \text{ sec.} \\ 2\mu - 1 & \text{for } 0.03 \text{ sec.} < T_n \le 0.5 \text{ sec.} \\ \mu & \text{for } T_n \ge 0.5 \text{ sec.} \end{cases} \tag{35.13}$$

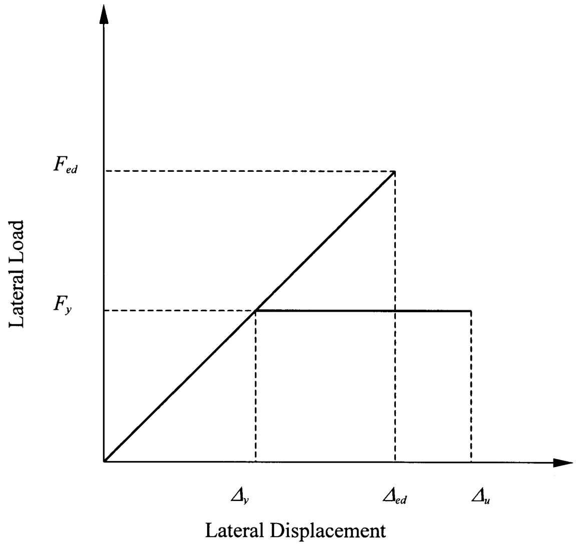

FIGURE 35.13 Lateral load–displacement relations.

For very short period ($T_n \le 0.03$ sec) in the acceleration-sensitive region, the elastic displacement demand Δ_{ed} is less than displacement capacity Δ_u (see Figure 35.13). The reduction factor $f(\mu) = 1$ implies that the structure should be designed and remained at elastic to avoid excessive inelastic deformation. For intermediate period (0.03 sec < $T_n \le 0.5$ sec) in the velocity-sensitive region, elastic displacement demand Δ_{ed} may be greater or less than displacement capacity Δ_u and the reduction factor is based on the equal-energy concept. For the very long period ($T_n > 0.5$ sec) in the displacement-sensitive region, the reduction factor is based on the equal displacement concept.

35.2.5 Example of an SDOF system

Given

An SDOF bridge structure is shown in Figure 35.14. To simplify the problem, the bridge is assumed to move only in the longitudinal direction. The total resistance against the longitudinal motion comes in the form of friction at bearings and this could be considered a damper. Assume the following properties for the structure: damping ratio $\xi = 0.05$, area of superstructure $A = 3.57$ m^2, moment of column $I_c = 0.1036$ m^4, E_c of column = 20,700 MPa, material density $\rho = 2400$ kg/m^3, length of column $L_c = 9.14$ m, and length of the superstructure $L_s = 36.6$ m. The acceleration response curve of the structure is given in the Figure 35.11. Determine (1) natural period of the structure, (2) damped period of the structure, (3) maximum displacement of the superstructure, and (4) maximum moment in the column.

Solution

$$\text{Stiffness:}\quad k = \frac{12E_cI_c}{L_c^3} = \frac{12(20700\times10^6)(0.1036)}{9.14^3} = 33690301 \text{ N/m}$$

$$\text{Mass:}\quad m = AL_s\rho = (3.57)(36.6)(2400) = 313{,}588.8 \text{ kg}$$

$$\text{Natural circular frequency:}\quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{33{,}690{,}301}{313{,}588.8}} = 10.36 \text{ rad/s}$$

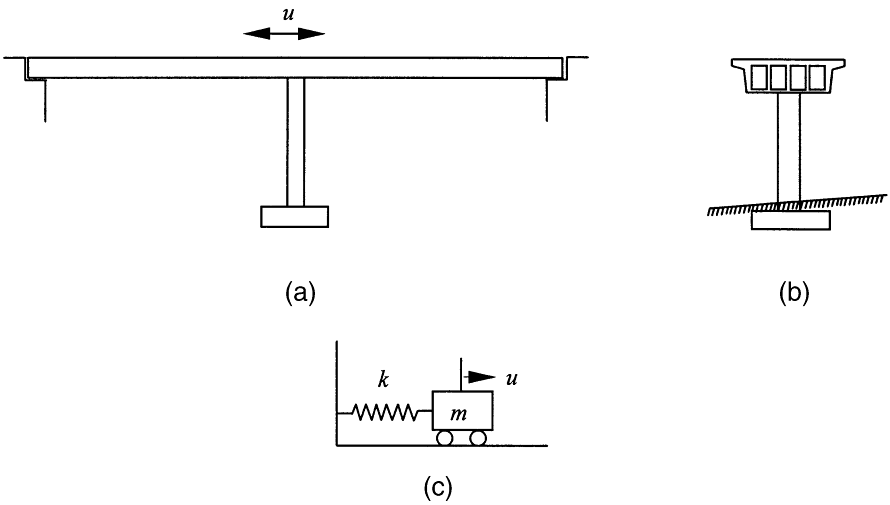

FIGURE 35.14 SDOF bridge example. (a) Two-span bridge schematic diagram; (b) single column bent; (c) idealized equivalent model for longitudinal response.

$$\text{Natural cyclic frequency: } f_n = \frac{\omega_n}{2\pi} = \frac{10.36}{2\pi} = 1.65 \text{ cycles/s}$$

$$\text{Natural period of the structure: } T_n = \frac{1}{f_n} = \frac{1}{1.65} = 0.606 \text{ s}$$

The *damped* circular frequency is given by

$$\omega_d = \omega_n \sqrt{1-\xi^2} = 10.36\sqrt{1-0.05^2} = 10.33 \text{ rad/s}$$

The *damped* period of the structure is given by

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{10.33} = 0.608 \text{ s}$$

From the ARS curve, for a period of 0.606 s, the maximum acceleration of the structure will be 0.9 g = 1.13 × 9.82 = 11.10 m/s. Then,

$$\text{The force acting on the mass} = m \times 11.10 = 313588.8 \times 11.10 = 3.48 \text{MN}$$

$$\text{The maximum displacement } = \frac{FL^3{}_c}{12EI_c} = \frac{3.48 \times 9.14^3}{12 \times 20700 \times 0.1036} = 0.103 \text{m}$$

$$\text{The maximum moment in the column } = \frac{FL_c}{2} = \frac{3.48 \times 9.14}{2} = 15.90 \text{ MN-m}$$

35.3 Multidegree-of-Freedom System

The SDOF approach may not be applicable for complex structures such as multilevel frame structure and bridges with several supports. To predict the response of a complex structure, the structure is discretized with several members of lumped masses. As the number of lumped masses increases, the number of displacements required to define the displaced positions of all masses increases. The response of a multidegree of freedom (MDOF) system is discussed in this section.

35.3.1 Equation of Motion

The equation of motion of an MDOF system is similar to the SDOF system, but the stiffness **k**, mass **m**, and damping **c** are matrices. The equation of motion to an MDOF system under ground motion can be written as

$$[\mathbf{M}]\{\ddot{u}\}+[\mathbf{C}]\{\dot{u}\}+[\mathbf{K}]\{u\}=-[\mathbf{M}]\,\{\mathbf{B}\}\ddot{u}_g \tag{35.14}$$

The stiffness matrix $[\mathbf{K}]$ can be obtained from standard static displacement-based analysis models and may have off-diagonal terms. The mass matrix $[\mathbf{M}]$ due to the negligible effect of mass coupling can best be expressed in the form of tributary lumped masses to the corresponding displacement degree of freedoms, resulting in a diagonal or uncoupled mass matrix. The damping matrix $[\mathbf{C}]$ accounts for all the energy-dissipating mechanisms in the structure and may have off-diagonal terms. The vector $\{\mathbf{B}\}$ is a displacement transformation vector that has values 0 and 1 to define degrees of freedoms to which the earthquake loads are applied.

35.3.2 Free Vibration and Vibration Modes

To understand the response of MDOF systems better, we look at the *undamped, free* vibration of an N degrees of freedom (N-DOF) system first.

Undamped Free Vibration

By setting $[\mathbf{C}]$ and $\ddot{u}_g$ to zero in the Eq. (35.14), the equation of motion of undamped, free vibration of an N-DOF system can be shown as:

$$[\mathbf{M}]\{\ddot{u}\}\ +[\mathbf{K}]\{u\}=0 \tag{35.15}$$

where $[\mathbf{M}]$ and $[\mathbf{K}]$ are $n \times n$ square matrices.

Equation (35.15) could then be rearranged to

$$\Big[[\mathbf{K}]-\omega_n^{\,2}[\mathbf{M}]\Big]\{\phi_n\}=0 \tag{35.16}$$

where $\{\phi_n\}$ is the deflected shape matrix. Solution to this equation can be obtained by setting

$$\Big|[\mathbf{K}]-\omega_n^{\,2}[\mathbf{M}]\Big|=0 \tag{35.17}$$

The roots or eigenvalues of Eq. (35.17) will be the N natural frequencies of the dynamic system. Once the natural frequencies (ω_n) are estimated, Eq. (35.16) can be solved for the corresponding N independent, deflected shape matrices (or eigenvectors), $\{\phi_n\}$. In other words, a vibrating system with N-DOFs will have N natural frequencies (usually arranged in sequence from smallest to largest), corresponding N natural periods T_n, and N natural mode shapes $\{\phi_n\}$. These eigenvectors are sometimes referred to as natural modes of vibration or natural mode shapes of vibration. It is important to recognize that the eigenvectors or mode shapes represent only the deflected shape corresponding to the natural frequency, not the actual deflection magnitude.

The N eigenvectors can be assembled in a single $n \times n$ square matrix $[\boldsymbol{\Phi}]$, modal matrix, where each column represents the coefficients associated with the natural mode. One of the important aspects of these mode shapes is that they are orthogonal to each other. Stated mathematically,

$$\text{If} \quad \omega_n \neq \omega_r, \qquad \{\phi_n\}^T [K]\{\phi_r\} = 0 \quad \text{and} \quad \{\phi_n\}^T [M]\{\phi_r\} = 0 \tag{35.18}$$

$$\left[K^*\right] = [\Phi]^T [K]\,[\Phi] \tag{35.19}$$

$$\left[M^*\right] = [\Phi]^T [M]\,[\Phi] \tag{35.20}$$

where $[\mathbf{K}]$ and $[\mathbf{M}]$ have off-diagonal elements, whereas $\left[\mathbf{K}^*\right]$ and $\left[\mathbf{M}^*\right]$ are diagonal matrices.

Damped Free Vibration

When damping of the MDOF system is included, the free vibration response of the damped system will be given by

$$[\mathbf{M}]\{\ddot{u}\} + [\mathbf{C}]\{\dot{u}\} + [\mathbf{K}]\{u\} = 0 \tag{35.21}$$

The displacements are first expressed in terms of natural mode shapes, and later they are multiplied by the transformed natural mode matrix to obtain the following expression:

$$\left[\mathbf{M}^*\right]\{\ddot{Y}\} + \left[\mathbf{C}^*\right]\{\dot{Y}\} + \left[\mathbf{K}^*\right]\{Y\} = 0 \tag{35.22}$$

where, $\left[\mathbf{M}^*\right]$ and $\left[\mathbf{K}^*\right]$ are diagonal matrices given by Eqs. (35.19) and (35.20) and

$$\left[\mathbf{C}^*\right] = [\boldsymbol{\Phi}]^T [\mathbf{C}]\,[\boldsymbol{\Phi}] \tag{35.23}$$

While $\left[\mathbf{M}^*\right]$ and $\left[\mathbf{K}^*\right]$ are diagonal matrices, $\left[\mathbf{C}^*\right]$ may have off diagonal terms. When $\left[\mathbf{C}^*\right]$ has off diagonal terms, the damping matrix is referred to as a *nonclassical* or *nonproportional* damping matrix. When $\left[\mathbf{C}^*\right]$ is diagonal, it is referred to as a *classical* or *proportional* damping matrix. Classical damping is an appropriate idealization when similar damping mechanisms are distributed throughout the structure. Nonclassical damping idealization is appropriate for the analysis when the damping mechanisms differ considerably within a structural system.

Since most bridge structures have predominantly one type of construction material, bridge structures could be idealized as a classical damping structural system. Thus, the damping matrix of Eq. (35.22) will be a diagonal matrix for most bridge structures. And, the equation of nth mode shape or generalized nth modal equation is given by

$$\ddot{Y}_n + 2\xi_n \omega_n \dot{Y}_n + \omega^2\, Y_n = 0 \tag{35.24}$$

Equation (35.24) is similar to the Eq. (35.7) of an SDOF system. Also, the vibration properties of each mode can be determined by solving the Eq. (35.24).

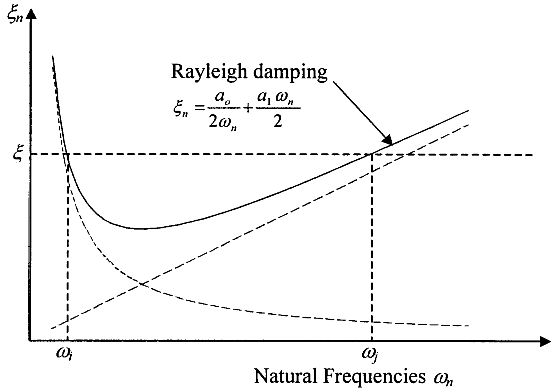

FIGURE 35.15 Rayleigh damping variation with natural frequency.

Rayleigh Damping

The damping of a structure is related to the amount of energy dissipated during its motion. It could be assumed that a portion of the energy is lost due to the deformations, and thus damping could be idealized as proportional to the stiffness of the structure. Another mechanism of energy dissipation could be attributed to the mass of the structure, and thus damping idealized as proportional to the mass of the structure. In Rayleigh damping, it is assumed that the damping is proportional to the mass and stiffness of the structure.

$$[\mathbf{C}] = a_o[\mathbf{M}] + a_1[\mathbf{K}] \tag{35.25}$$

The generalized damping of the n^{th} mode is then given by

$$C_n = a_o M_n + a_1 K_n \tag{35.26}$$

$$C_n = a_o M_n + a_1 \omega_n^2 M_n \tag{35.27}$$

$$\xi_n = \frac{C_n}{2M_n\omega_n} \tag{35.28}$$

$$\xi_n = \frac{a_o}{2}\frac{1}{\omega_n} + \frac{a_1}{2}\omega_n \tag{35.29}$$

Figure 35.15 shows the Rayleigh damping variation with natural frequency. The coefficients a_o and a_1 can be determined from specified damping ratios at two independent dominant modes (say, i^{th} and j^{th} modes). Expressing Eq. (35.29) for these two modes will lead to the following equations:

$$\xi_i = \frac{a_o}{2}\frac{1}{\omega_i} + \frac{a_1}{2}\omega_i \tag{35.30}$$

$$\xi_j = \frac{a_o}{2}\frac{1}{\omega_j} + \frac{a_1}{2}\omega_j \tag{35.31}$$

When the damping ratio at both the i^{th} and j^{th} modes is the same and equals ξ, it can be shown that

$$a_o = \xi \frac{2\omega_i \omega_j}{\omega_i + \omega_j} \qquad a_1 = \xi \frac{2}{\omega_i + \omega_j} \tag{35.32}$$

It is important to note that the damping ratio at a mode between the i^{th} and j^{th} mode is less than ξ. And, in practical problems the specified damping ratios should be chosen to ensure reasonable values in all the mode shapes that lie between the *i*th and *j*th mode shapes.

35.3.3 Modal Analysis and Modal Participation Factor

In previous sections, we have discussed the basic vibration properties of an MDOF system. Now, we will look at the response of an MDOF system to earthquake ground motion. The basic equation of motion of the MDOF for an earthquake ground motion given by Eq. (35.14) is repeated here:

$$[\mathbf{M}]\{\ddot{u}\} + [\mathbf{C}]\{\dot{u}\} + [\mathbf{K}]\{u\} = -[\mathbf{M}]\,\{B\}\ddot{u}_g$$

The displacement is first expressed in terms of natural mode shapes, and later it is multiplied by the transformed natural mode matrix to obtain the following expression:

$$\left[\mathbf{M}^*\right]\left\{\ddot{Y}\right\} + \left[\mathbf{C}^*\right]\left\{\dot{Y}\right\} + \left[\mathbf{K}^*\right]\{Y\} = -\,[\mathbf{\Phi}]^T[\mathbf{M}]\{B\}\,\ddot{u}_g \tag{35.33}$$

And, the equation of the n^{th} mode shape is given by

$$M_n^*\,\ddot{Y}_n + 2\xi_n\,\omega_n\,M_n^*\dot{Y}_n + \omega^2 M_n^*\,Y_n = L_n\,\ddot{u}_g \tag{35.34}$$

where

$$M_n^* = \left\{\phi_n\right\}^T[\mathbf{M}]\left\{\phi_n\right\} \tag{35.35}$$

$$L_n = -\left\{\phi_n\right\}^T[\mathbf{M}]\,[\mathbf{B}] \tag{35.36}$$

The L_n is referred to as the *modal participation factor* of the *n*th mode.

By dividing the Eq. (35.34) by M_n^*, the generalized modal equation of the *n*th mode becomes

$$\ddot{Y}_n + 2\xi_n\,\omega_n\,\dot{Y}_n + \omega^2\,Y_n = \left(\frac{L_n}{M_n^*}\right)\ddot{u}_g \tag{35.37}$$

Equation (35.34) is similar to the equation motion of an SDOF system, and thus Y_n can be determined by using methods similar to those described for SDOF systems. Once Y_n is established, the displacement due to the n^{th} mode will be given by $u_n(t) = \phi_n Y_n(t)$. The total displacement due to combination of all mode shapes can then be determined by summing up all displacements for each mode and is given by

$$u(t) = \sum \phi_n Y_n(t) \tag{35.38}$$

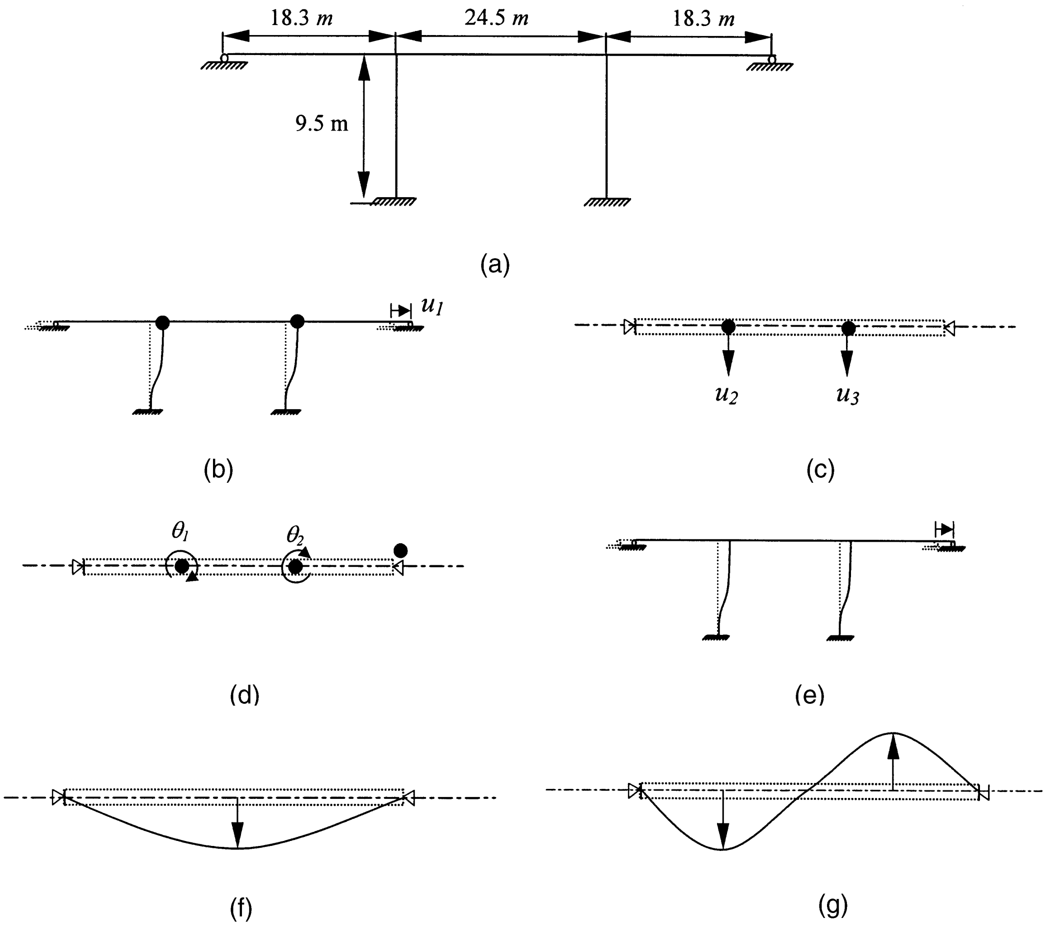

FIGURE 35.16 Three-span continuous framed bridge structure of MDOF example. (a) Schematic diagram; (b) longitudinal degree of freedom; (c) transverse degree of freedom; (d) rotational degree of freedom; (e) mode shape 1; (f) mode shape 2; (g) mode shape 3.

This approach is sometimes referred to as the classical mode superposition method. Similar to the estimation of the total displacement, the element forces can also be estimated by adding the element forces for each mode shape.

35.3.4 Example of an MDOF System

Given

The bridge shown in Figure 35.16 is a three-span continuous frame structure. Details of the bridge are as follows: span lengths are 18.3, 24.5, and 18.3 m.; column length is 9.5 m; area of superstructure is 5.58 m^2; moment of inertia of superstructure is 70.77 m^4; moment of inertia of column is 0.218 m^4; modulus of elasticity of concrete is 20,700 MPa. Determine the vibration modes and frequencies of the bridge.

Solution

As shown in Figures 35.16b, c, and d, five degrees of freedom are available for this structure. Stiffness and mass matrices are estimated separately and the results are given here.

$$[\mathbf{K}] = \begin{bmatrix} 126318588 & 0 & 0 & 0 & 0 \\ 0 & 1975642681 & -1194370500 & -1520122814 & -14643288630 \\ 0 & -1194370500 & 1975642681 & 14643288630 & 1520122814 \\ 0 & -1520122814 & 14643288630 & 479327648712 & 119586857143 \\ 0 & -14643288630 & 1520122814 & 119586857143 & 479327648712 \end{bmatrix}$$

$$[\mathbf{M}] = \begin{bmatrix} 81872 & 0 & 0 & 0 & 0 \\ 0 & 286827 & 0 & 0 & 0 \\ 0 & 0 & 286827 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Condensation procedure will eliminate the rotational degrees of freedom and will result in three degrees of freedom. (The condensation procedure is performed separately and the result is given here.) The equation of motion of free vibration of the structure is

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$

Substituting condensed stiffness and mass matrices into the above equation gives

$$\begin{bmatrix} 81872 & 0 & 0 \\ 0 & 286827 & 0 \\ 0 & 0 & 286827 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} 126318588 & 0 & 0 \\ 0 & 1975642681 & -1194370500 \\ 0 & -1194370500 & 1975642681 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

The above equation can be rearranged in the following form:

$$\frac{1}{\omega^2}[M]^{-1}[K]\{\phi\} = \{\phi\}$$

Substitution of appropriate values in the above expression gives the following

$$\frac{1}{\omega_n^2} \begin{bmatrix} \frac{1}{818172} & 0 & 0 \\ 0 & \frac{1}{286827} & 0 \\ 0 & 0 & \frac{1}{286827} \end{bmatrix} \begin{bmatrix} 126318588 & 0 & 0 \\ 0 & 1518171572 & -1215625977 \\ 0 & -1215625977 & 1518171572 \end{bmatrix} \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \phi_{3n} \end{Bmatrix} = \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \phi_{3n} \end{Bmatrix}$$

$$\frac{1}{\omega_n^2} \begin{bmatrix} 154.39 & 0 & 0 \\ 0 & 5292.9 & -4238.2 \\ 0 & -4238.2 & 5292.9 \end{bmatrix} \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \phi_{3n} \end{Bmatrix} = \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \phi_{3n} \end{Bmatrix}$$

By assuming different vibration modes, natural frequencies of the structure can be estimated.

Substitution of vibration mode $\{1\ 0\ 0\}^T$ will result in the first natural frequency.

$$\frac{1}{\omega_n^2}\begin{bmatrix}154.39 & 0 & 0\\ 0 & 5292.9 & -4238.2\\ 0 & -4238.2 & 5292.9\end{bmatrix}\begin{Bmatrix}1\\0\\0\end{Bmatrix} = \frac{1}{\omega_n^2}\begin{bmatrix}154.39\\0\\0\end{bmatrix} = \begin{Bmatrix}1\\0\\0\end{Bmatrix}$$

Thus, $\omega_n^2 = 154.39$ and $\omega_n = 12.43$ rad / s

By substituting the vibration modes of $\{0\ 1\ 1\}^T$ and $\{0\ \ 1\ -1\}^T$ in the above expression, the other two natural frequencies are estimated as 32.48 and 97.63 rad/s.

35.3.5 Multiple-Support Excitation

So far we have assumed that all supports of a structural system undergo the same ground motion. This assumption is valid for structures with foundation supports close to each other. However, for long-span bridge structures, supports may be widely spaced. As described in Section 35.1.2, earth motion at a location depends on the localized soil layer and the distance from the epicenter. Thus, bridge structures with supports that lie far from each other may experience different earth excitation. For example, Figure 35.17c, d, and e shows the predicted earthquake motions at Pier W3 and Pier W6 of the San Francisco–Oakland Bay Bridge (SFOBB) in California. The distance between Pier W3 and Pier W6 of the SFOBB is approximately 1411 m. These excitations are predicted by the California Department of Transportation by considering the soil and rock properties in the vicinity of the SFOBB and expected Earth movements at the San Andreas and Hayward faults. Note that the Earth motion at Pier W3 and Pier W6 are very different. Furthermore, Figures 35.17c, d, and e indicates that the Earth motion not only varies with the location, but also varies with direction. Thus, to evaluate the response of long, multiply supported, and complicated bridge structures, use of the actual earthquake excitation at each support is recommended.

The equation of motion of a multisupport excitation would be similar to Eq. (35.14), but the only difference is now that $\{B\}\ddot{u}_g$ is replaced by an displacement array $\{\ddot{u}_g\}$. And, the equation of motion for the multisupport system becomes

$$[\mathbf{M}]\{\ddot{u}\} + [\mathbf{C}]\{\dot{u}\} + [\mathbf{K}]\{u\} = -[\mathbf{M}]\{\ddot{u}_g\} \tag{35.39}$$

where $\{\ddot{u}_g\}$ has the acceleration at each support locations and has zero value at nonsupport locations. By using the uncoupling procedure described in the previous sections, the modal equation of the n^{th} mode can be written as

$$\ddot{Y}_n + 2\xi_n\omega_n\dot{Y}_n + \omega^2 Y_n = -\sum_{l=1}^{N_g}\frac{L_n}{M_n^*}\ddot{u}_g \tag{35.40}$$

where N_g is the total number of externally excited supports.

The deformation response of the n^{th} mode can then be determined as described in previous sections. Once the displacement responses of the structure for all the mode shapes are estimated, the total dynamic response can be obtained by combining the displacements.

35.3.6 Time History Analysis

When the structure enters the nonlinear range, or has nonclassical damping properties, modal analysis cannot be used. A numerical integration method, sometimes referred to as time history analysis, is required to get more accurate responses of the structure.

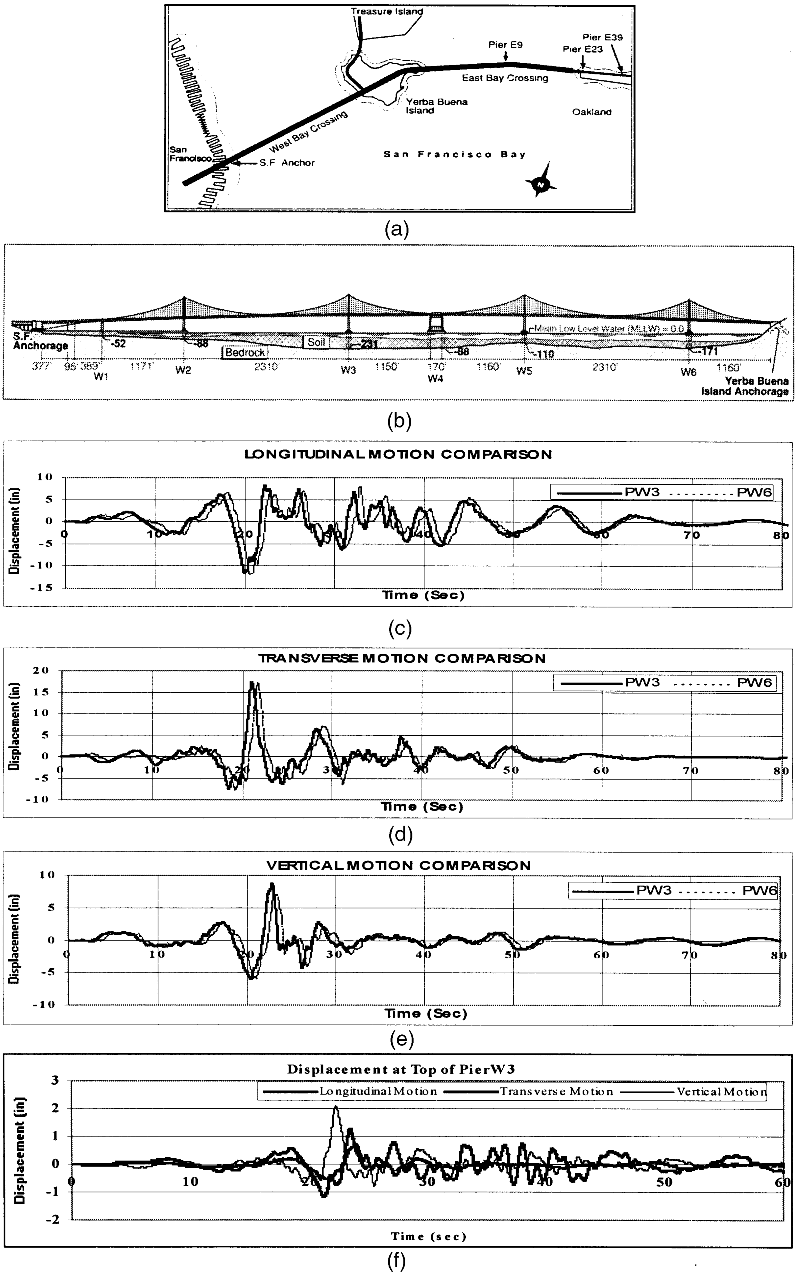

FIGURE 35.17 San Francisco–Oakland Bay Bridge. (a) Vicinity map; (b) general plan elevation; (c) longitudinal motion at rock level; (d) transverse motion at rock level; (e) vertical motion at rock level; (f) displacement response at top of Pier W3.

In a time history analysis, the timescale is divided into a series of smaller steps, d_τ. Let us say the response at i^{th} time interval has already determined and is denoted by $u_i, \dot{u}_i, \ddot{u}_i$. Then, the response of the system at i^{th} time interval will satisfy the equation of motion (Eq. 35.39).

$$[\mathbf{M}]\{\ddot{u}_i\} + [\mathbf{C}]\{\dot{u}_i\} + [\mathbf{K}]\{u_i\} = -[\mathbf{M}]\{\ddot{u}_{gi}\} \tag{35.41}$$

The time-stepping method enables us to step ahead and determine the responses $u_{i+1}, \dot{u}_{i+1}, \ddot{u}_{i+1}$ at the $i+1^{th}$ time interval by satisfying Eq. (35.39). Thus, the equation of motion at $i+1^{th}$ time interval will be

$$[\mathbf{M}]\{\ddot{u}_{i+1}\}+[\mathbf{C}]\{\dot{u}_{i+1}\}+[\mathbf{K}]\{u_{i+1}\}=-[\mathbf{M}]\{\ddot{u}_{gi+1}\} \tag{35.42}$$

Equation (35.42) needs to be solved prior to proceeding to the next time step. By stepping through all the time steps, the actual response of the structure can be determined at all time instants.

Example of Time History Analysis

The Pier W3 of the SFOBB was modeled using the ADINA [13] program and nonlinear analysis was performed using the displacement time histories. The displacement time histories in three directions are applied at the bottom of the Pier W3 and the response of the Pier W3 was studied to estimate the demand on Pier W3. One of the results, the displacement response at top of Pier W3, is shown in Figure 35.17f.

35.4 Response Spectrum Analysis

Response spectrum analysis is an approximate method of dynamic analysis that gives the maximum response (acceleration, velocity, or displacement) of an SDOF system with the same damping ratio, but with different natural frequencies, respond to a specified seismic excitation. Structural models with n degrees of freedom can be transformed to n single-degree systems and response spectra principles can be applied to systems with many degrees of freedom. For most ordinary bridges, a complete time history is not required. Because the design is generally based on the maximum earthquake response, response spectrum analysis is probably the most common method used in design offices to determine the maximum structural response due to transient loading. In this section, we will discuss basic procedures of response spectrum analysis for bridge structures.

35.4.1 Single-Mode Spectral Analysis

Single-mode spectral analysis is based on the assumption that earthquake design forces for structures respond predominantly in the first mode of vibration. This method is most suitable to regular linear elastic bridges to compute the forces and deformations, but is not applicable to irregular bridges (unbalanced spans, unequal stiffness in the columns, etc.) because higher modes of vibration affect the distribution of the forces and resulting displacements significantly. This method can be applied to both continuous and noncontinuous bridge superstructures in either the longitudinal or transverse direction. Foundation flexibility at the abutments can be included in the analysis.

Single-mode analysis is based on Rayleigh's energy method — an approximate method which assumes a vibration shape for a structure. The natural period of the structure is then calculated by equating the maximum potential and kinetic energies associated with the assumed shape. The inertial forces $p_e(x)$ are calculated using the natural period, and the design forces and displacements are then computed using static analysis. The detailed procedure can be described in the following steps:

1. Apply uniform loading p_o over the length of the structure and compute the corresponding static displacements $u_s(x)$. The structure deflection under earthquake loading, $u_s(x,t)$ is then approximated by the shape function, $u_s(x)$, multiplied by the generalized amplitude function, $u(t)$, which satisfies the geometric boundary conditions of the structural system. This dynamic deflection is shown as

$$u(x,t) = u_s(x)\ u(t) \tag{35.43}$$

2. Calculate the generalized parameters $\alpha, \beta,$ and γ using the following equations:

$$\alpha = \int u_s(x)\,dx \tag{35.44}$$

$$\beta = \int w(x)\,u_s(x)\;dx \tag{35.45}$$

$$\gamma = \int w(x)\left[u_s(x)\right]^2 dx \tag{35.46}$$

where $w(x)$ is the weight of the dead load of the bridge superstructure and tributary substructure.

3. Calculate the period T_n

$$T_n = 2\pi\sqrt{\frac{\gamma}{p_o g\alpha}} \tag{35.47}$$

where g is acceleration of gravity (mm/s^2).

4. Calculate the static loading $p_e(x)$ which approximates the inertial effects associated with the displacement $u_s(x)$ using the ARS curve or the following equation [4]:

$$p_e(x) = \frac{\beta\, C_{sm}}{\gamma}\; w(x)\; u_s(x) \tag{35.48}$$

$$C_{sm} = \frac{1.2AS}{T_m^{\;2/3}} \tag{35.49}$$

where C_{sm} is the dimensionless elastic seismic response coefficient; A is the acceleration coefficient from the acceleration coefficient map; S is the dimensionless soil coefficient based on the soil profile type; T_n is the period of the structure as determined above; $p_e(x)$ is the intensity of the equivalent static seismic loading applied to represent the primary mode of vibration (N/mm).

5. Apply the calculated loading $p_e(x)$ to the structure as shown in the Figure 35.18 and compute the structure deflections and member forces.

This method is an iterative procedure, and the previous calculations are used as input parameters for the new iteration leading to a new period and deflected shape. The process is continued until the assumed shape matches the fundamental mode shape.

35.4.2 Uniform-Load Method

The uniform-load method is essentially an equivalent static method that uses the uniform lateral load to compute the effect of seismic loads. For simple bridge structures with relatively straight alignment, small skew, balanced stiffness, relatively light substructure, and with no hinges, the uniform-load method may be applied to analyze the structure for seismic loads. This method is not suitable for bridges with stiff substructures such as pier walls. This method assumes continuity of the structure and distributes earthquake force to all elements of the bridge and is based on the fundamental mode of vibration in either a longitudinal or transverse direction [5]. The period of

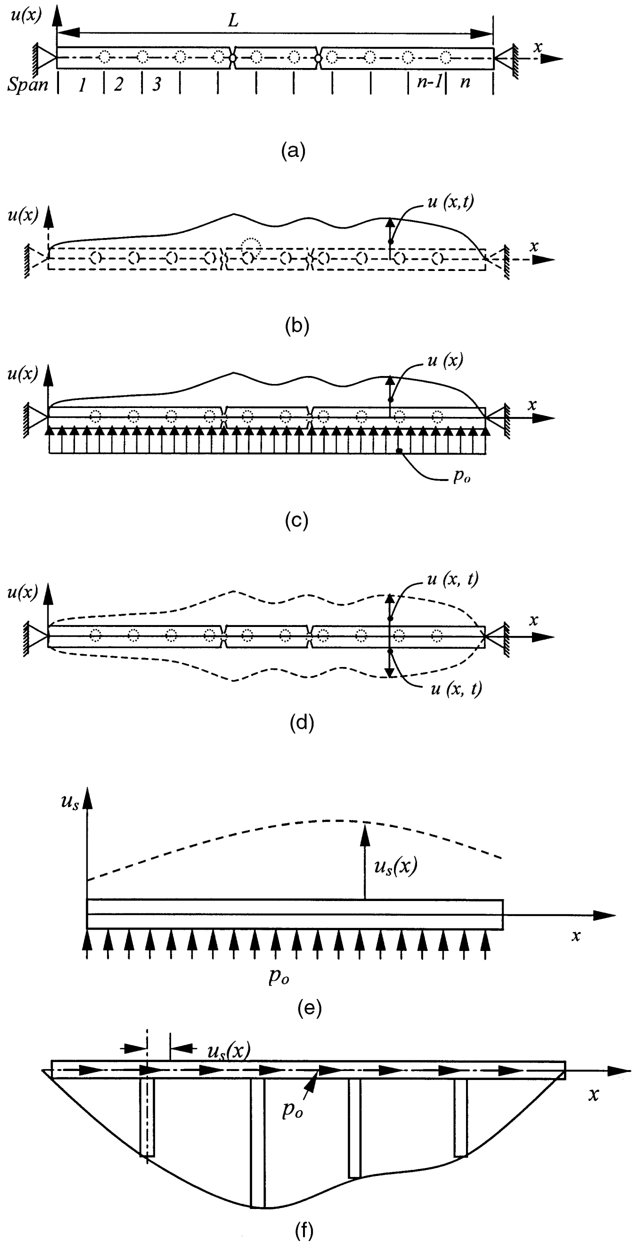

FIGURE 35.18 Single-mode spectral analysis method. (a) Plan view of a bridge subjected to transverse earthquake motion. (b) Displacement function describing the transverse position of the bridge deck. (c) Deflected shape due to uniform static loading. (d) Transverse free vibration of the bridge in assumed mode shape. (e) Transverse loading (f) longitudinal loading.

vibration is taken as that of an equivalent single mass–spring oscillator. The maximum displacement that occurs under the arbitrary uniform load is used to calculate the stiffness of the equivalent spring. The seismic elastic response coefficient C_{sm} or the ARS curve is then used to calculate the equivalent uniform seismic load, using which the displacements and forces are calculated. The following steps outline the uniform load method:

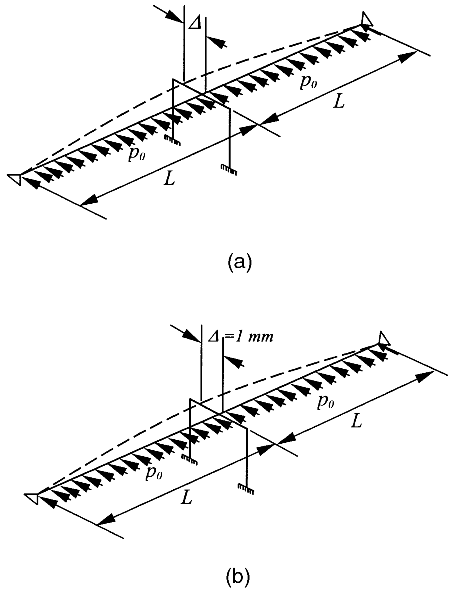

FIGURE 35.19 Structure idealization and deflected shape for uniform load method. (a) Structure idealization; (b) deflected shape with maximum displacement of 1 mm.

1. Idealize the structure into a simplified model and apply a uniform horizontal load (p_o) over the length of the bridge as shown in Figure 35.19. It has units of force/unit length and may be arbitrarily set equal to 1 N/mm.
2. Calculate the static displacements $u_s(x)$ under the uniform load p_o using static analysis.
3. Calculate the maximum displacement $u_{s,\max}$ and adjust it to 1 mm by adjusting the uniform load p_o.
4. Calculate bridge lateral stiffness K using the following equation:

$$K = \frac{p_o L}{u_{s,\max}} \tag{35.50}$$

   where L is total length of the bridge (mm); and $u_{s,\max}$ is maximum displacement (mm).
5. Calculate the total weight W of the structure including structural elements and other relevant loads such as pier walls, abutments, columns, and footings, by

$$W = \int w(x)dx \tag{35.51}$$

   where $w(x)$ is the nominal, unfactored dead load of the bridge superstructure and tributary substructure.
6. Calculate the period of the structure T_n using the following equation:

$$T_n = \frac{2\pi}{31.623}\sqrt{\frac{W}{gK}} \tag{35.52}$$

   where g is acceleration of gravity (m/s²).

7. Calculate the equivalent static earthquake force p_e using the ARS curve or using the following equation:

$$p_e = \frac{C_{sm} W}{L} \tag{35.53}$$

8. Calculate the structure deflections and member forces by applying p_e to the structure.

35.4.4 Multimode Spectral Analysis

The multimode spectral analysis method is more sophisticated than single-mode spectral analysis and is very effective in analyzing the response of more complex linear elastic structures to an earthquake excitation. This method is appropriate for structures with irregular geometry, mass, or stiffness. These irregularities induce coupling in three orthogonal directions within each mode of vibration. Also, for these bridges, several modes of vibration contribute to the complete response of the structure. A multimode spectral analysis is usually done by modeling the bridge structure consisting of three-dimensional frame elements with structural mass lumped at various locations to represent the vibration modes of the components. Usually, five elements per span are sufficient to represent the first three modes of vibration. A general rule of thumb is, to capture the i^{th} mode of vibration, the span should have at least $(2i-1)$ elements. For long-span structures many more elements should be used to capture all the contributing modes of vibration. To obtain a reasonable response, the number of modes should be equal to at least three times the number of spans. This analysis is usually performed with a dynamic analysis computer program such as ADINA [13], GTSTRUDL [14], SAP2000 [15], ANSYS [16], and NASTRAN [17]. For bridges with outrigger bents, C-bents, and single column bents, rotational moment of inertia of the superstructure should be included. Discontinuities at the hinges and abutments should be included in the model. The columns and piers should have intermediate nodes at quarter points in addition to the nodes at the ends of the columns.

By using the programs mentioned above, frequencies, mode shapes, member forces, and joint displacements can be computed. The following steps summarize the equations used in the multimode spectral analysis [5].

1. Calculate the dimensionless mode shapes $\{\phi_i\}$ and corresponding frequencies ω_i by

$$\left[[\mathbf{K}] - \omega^2 [\mathbf{M}] \right] \{u\} = 0 \tag{35.54}$$

where

$$u_i = \sum_{j=1}^{n} \phi_j y_j = \Phi y_i \tag{35.55}$$

$y_j =$ modal amplitude of jth mode; $\phi_j =$ shape factor of j^{th} mode; Φ = mode-shape matrix. The periods for i^{th} mode can then be calculated by

$$T_i = \frac{2\pi}{\omega_i} \quad (i = 1, 2, \ldots, n) \tag{35.56}$$

2. Determine the maximum absolute mode amplitude for the entire time history is given by

$$Y_i(t)_{\max} = \frac{T_i^2 S_a(\xi_i, T_i)}{4\pi^2} \frac{\{\phi_i\}^T [\mathbf{M}]\{B\}\ddot{u}_g}{\{\phi_i\}^T [\mathbf{M}]\{\phi_i\}} \tag{35.57}$$

   where $S_a(\xi_i, T_i) = gC_{sm}$ is the acceleration response spectral value; C_{sm} is the elastic seismic response coefficient for mode $m = 1.2AS/T_n^{2/3}$; A is the acceleration coefficient from the acceleration coefficient map; S is the dimensionless soil coefficient based on the soil profile type; T_n is the period of the n^{th} mode of vibration.
3. Calculate the value of any response quantity $Z(t)$ (shear, moment, displacement) using the following equation:

$$Z(t) = \sum_{i=1}^{n} A_i Y_i(t) \tag{35.58}$$

   where coefficients A_i are functions of mode shape matrix (Φ) and force displacement relationships.
4. Compute the maximum value of $Z(t)$ during an earthquake using the mode combination methods described in the next section.

Modal Combination Rules

The mode combination method is a very useful tool for analyzing bridges with a large number of degrees of freedom. In a linear structural system, maximum response can be estimated by mode combination after calculating natural frequencies and mode shapes of the structure using free vibration analysis. The maximum response cannot be computed by adding the maximum response of each mode because different modes attain their maximum values at different times. The absolute sum of the individual modal contributions provides an upper bound which is generally very conservative and not recommended for design. There are several different empirical or statistical methods available to estimate the maximum response of a structure by combining the contributions of different modes of vibrations in a spectral analysis. Two commonly used methods are the square root of sum of squares (SRSS) and the complete quadratic combination (CQC).

For an undamped structure, the results computed using the CQC method are identical to those using the SRSS method. For structures with closely spaced dominant mode shapes, the CQC method is precise whereas SRSS estimates inaccurate results. Closely spaced modes are those within 10% of each other in terms of natural frequency. The SRSS method is suitable for estimating the total maximum response for structures with well-spaced modes. Theoretically, all mode shapes must be included to calculate the response, but fewer mode shapes can be used when the corresponding mass participation is over 85% of the total structure mass. In general, the factors considered to determine the number of modes required for the mode combination are dependent on the structural characteristics of the bridge, the spatial distribution, and the frequency content of the earthquake loading. The following list [14] summarizes several commonly used mode combination methods to compute the maximum total response. The variable Z represents the maximum value of some response quantity (displacement, shear, etc.), Z_i is the peak value of that quantity in the i^{th} mode, and N is the total number of contributing modes.

1. *Absolute Sum*: The absolute sum is sum of the modal contributions:

$$Z = \sum_{i=1}^{N} |Z_i| \tag{35.59}$$

2. *SRSS or Root Mean Square* (*RMS*) *Method*: This method computes the maximum by taking the square root of sum of squares of the modal contributions:

$$Z = \left[\sum_{i=1}^{N} Z_i^2 \right]^{1/2} \tag{35.60}$$

3. *Peak Root Mean Square* (*PRMS*): Absolute value of the largest modal contribution is added to the root mean square of the remaining modal contributions:

$$Z_j = |\max \ Z_i| \tag{35.61}$$

$$Z = \left[\sum_{i=1}^{N} Z_i^2 \right]^{1/2} + Z_j \quad \text{with} \quad i \neq j \tag{35.62}$$

4. *CQC*: Cross correlations between all modes are considered:

$$Z = \left[\sum_{i=1} \sum_{j=1} Z_i \ \rho_{ij} \ Z_j \right] \tag{35.63}$$

$$\rho_{ij} = \frac{8\sqrt{\xi_i \xi_j}\left(\xi_i + r\xi_j\right) r^{3/2}}{\left(1-r^2\right)^2 + 4\xi_i \xi_j r\left(1+r^2\right) + 4\left(\xi_i^2 + \xi_j^2\right) r^2} \tag{35.64}$$

where

$$r = \frac{\omega_j}{\omega_i} \tag{35.65}$$

5. *Nuclear Regulatory Commission Grouping Method*: This method is similar to RMS method with additional accounting for groups of modes whose frequencies are within 10%.

$$Z = \left[\sum_{i=1}^{N} Z_i^2 + \sum_{g=1}^{G} \sum_{n=s}^{e} \sum_{m=s}^{e} \left| Z_n^g \times Z_m^g \right| \right]^{1/2} \qquad n \neq m \tag{35.66}$$

where G is number of groups; s is mode shape number where the g^{th} group starts; e is mode shape number where the g^{th} group ends; and Z_i^g is the i^{th} modal contribution in the g^{th} group.

6. *Nuclear Regulatory Commission Ten Percent Method*: This method is similar to the RMS method with additional accounting for all modes whose frequencies are within 10%.

$$Z = \left[\sum_{i=1}^{N} Z_i^2 + 2 \sum \left| Z_n Z_m \right| \right]^{1/2} \tag{35.67}$$

The additional terms must satisfy

$$\frac{\omega_n - \omega_m}{\omega_m} \le 0.1 \qquad \text{for} \qquad 0.1 \le m \le n \le N \tag{35.68}$$

7. *Nuclear Regulatory Commission Double Sum Method*: This method is similar to the CQC method.

$$Z = \left[\sum_{i=1}^{N} \sum_{j=1}^{N} \left| Z_i Z_j \right| \quad \varepsilon_{ij} \right]^{1/2} \tag{35.69}$$

$$\varepsilon_{ij} = \left[1 + \left\{ \frac{\left(\omega_i' - \omega_j'\right)}{\left(\xi_i' \omega_i + \xi_j' \omega_j\right)} \right\} \right]^{-1} \tag{35.70}$$

$$\omega_i' = \omega\left[1 - \xi_i^2\right]^{1/2} \tag{35.71}$$

$$\xi_i' = \xi_i + \frac{2}{t_d \omega_i} \tag{35.72}$$

where t_d is the duration of support motion.

Combination Effects

Effects of ground motions in two orthogonal horizontal directions should be combined while designing bridges with simple geometric configurations. For bridges with long spans, outrigger bents, and with cantilever spans, or where effects due to vertical input are significant, vertical input should be included in the design along with two orthogonal horizontal inputs. When bridge structures are analyzed independently along each direction using response spectra analysis, then responses are combined either using methods, such as the SRSS combination rule as mentioned in the previous section, or using the alternative method described below. For structures designed using equivalent static analysis or modal analysis, seismic effects should be determined using the following alternative method for the following load cases:

1. *Seismic load case* 1: 100% Transverse + 30% Longitudinal + 30% Vertical
2. *Seismic load case* 2: 30% Transverse + 100% Longitudinal + 30% Vertical
3. *Seismic load case* 3: 30% Transverse + 30% Longitudinal + 100% Vertical

For structures designed using time-history analysis, the structure response is calculated using the input motions applied in orthogonal directions simultaneously. Where this is not feasible, the above alternative procedure can be used to combine the independent responses.

35.4.4 Multiple-Support Response Spectrum Method

Records from recent earthquakes indicate that seismic ground motions can significantly vary at different support locations for multiply supported long structures. When different ground motions are applied at various support points of a bridge structure, the total response can be calculated by superposition of responses due to independent support input. This analysis involves combination of dynamic response from single-input and pseudo-static response resulting from the motion of the supports relative to each other. The combination effects of dynamic and pseudo-static forces due to multiple support excitation on a bridge depend on the structural configuration of the bridge and the ground motion characteristics. Recently, Kiureghian et al. [7] presented a comprehensive study on the multiple-support response spectrum (MSRS) method based on fundamental principles of stationary random vibration theory for seismic analysis of multiply supported structures which accounts for the effects of variability between the support motions. Using the MSRS combination rule, the response of a linear structural system subjected to multiple support excitation can be computed directly in terms of conventional response spectra at the support degrees of freedom and a coherency function describing the spatial variability of the ground motion. This method accounts for the three important effects of ground motion spatial variability, namely, the incoherence effects, the wave passage effect, and the site response effect. These three components of ground motion spatial variability can strongly influence the response of multiply supported bridges and may amplify or deamplify the response by one order of magnitude. Two important limitations of this method are nonlinearities in the bridge structural components and/or connections and the effects of soil–structure interaction. This method is an efficient, accurate, and versatile solution and requires less computational time than a true time history analysis. Following are the steps that describe the MSRS analysis procedure.

1. *Determine the necessity of variable support motion analysis*: Three factors that influence the response of the structure under multiple support excitation are the distance between the supports of the structure, the rate of variability of the local soil conditions, and the stiffness of the structure. The first factor, the distance between the supports, influences the incoherence and wave passage effects. The second factor, the rate of variability of the local conditions, influences the site response. The third factor, the stiffness of the superstructure, plays an important role in determining the necessity of variable-support motion analysis. Stiff structures such as box-girder bridges may generate large internal forces under variable support motion, whereas flexible structures such as suspension bridges easily conform to the variable support motion.
2. *Determine the frequency response function for each support location.* Programs such as SHAKE [18] can be used to develop these functions using borehole data and time-domain site response analysis. Response spectra plots, peak ground displacements in three orthogonal directions for each support location, and a coherency function for each pair of degrees of freedom are required to perform the MSRS analysis. The comprehensive report by Kiureghian [7] provides all the formulas required to account for the effect of nonlinearity in the soil behavior and the site frequency involving the depth of the bedrock.
3. *Calculate the Structural Properties*: such as effective modal frequencies, damping ratios, influence coefficients and effective modal participation factors (ω_i, ξ_i, a_k, and b_{ki}) are to be computed externally and provided as input.
4. *Determine the response spectra plots, peak ground displacements in three directions, and a coherency function for each pair of support degrees of freedom required to perform MSRS analysis*: Three components of the coherency function are incoherence, wave passage effect, and site response effect. Analysis by an array of recordings is used to determine the incoherence component. The models for this empirical method are widely available [19]. Parameters such as shear wave velocity, the direction of propagation of seismic waves, and the angle of incidence are used to calculate the wave passage effect. The frequency response function determined in the previous steps is used to calculate the site response component.

35.5 Inelastic Dynamic Analysis

35.5.1 Equations of Motion

Inelastic dynamic analysis is usually performed for the safety evaluation of important bridges to determine the inelastic response of bridges when subjected to design earthquake ground motions.

Inelastic dynamic analysis provides a realistic measure of response because the inelastic model accounts for the redistribution of internal actions due to the nonlinear force displacement behavior of the components [20–25]. Inelastic dynamic analysis considers nonlinear damping, stiffness, load deformation behavior of members including soil, and mass properties. A step-by-step integration procedure is the most powerful method used for nonlinear dynamic analysis. One important assumption of this procedure is that acceleration varies linearly while the properties of the system such as damping and stiffness remain constant during the time interval. By using this procedure, a nonlinear system is approximated as a series of linear systems and the response is calculated for a series of small equal intervals of time Δt and equilibrium is established at the beginning and end of each interval.

The accuracy of this procedure depends on the length of the time increment Δt. This time increment should be small enough to consider the rate of change of loading $p(t)$, nonlinear damping and stiffness properties, and the natural period of the vibration. An SDOF system and its characteristics are shown in the Figure 35.20. The characteristics include spring and damping forces, forces acting on mass of the system, and arbitrary applied loading. The force equilibrium can be shown as

$$f_i(t) + f_d(t) + f_s(t) = p(t) \tag{35.73}$$

and the incremental equations of motion for time t can be shown as

$$m\,\Delta\ddot{u}(t) + c(t)\,\Delta\dot{u}(t) + k(t)\,\Delta u(t) = \Delta p(t) \tag{35.74}$$

Current damping $f_d(t)$, elastic forces $f_s(t)$ are then computed using the initial velocity $\dot{u}(t)$, displacement values $u(t)$, nonlinear properties of the system, damping $c(t)$, and stiffness $k(t)$ for that interval. New structural properties are calculated at the beginning of each time increment based on the current deformed state. The complete response is then calculated by using the displacement and velocity values computed at the end of each time step as the initial conditions for the next time interval and repeating until the desired time.

35.5.2 Modeling Considerations

A bridge structural model should have sufficient degrees of freedom and proper selection of linear/nonlinear elements such that a realistic response can be obtained. Nonlinear analysis is usually preceded by a linear analysis as a part of a complete analysis procedure to capture the physical and mechanical interactions of seismic input and structure response. Output from the linear response solution is then used to predict which nonlinearities will affect the response significantly and to model them appropriately. In other words, engineers can justify the effect of each nonlinear element introduced at the appropriate locations and establish the confidence in the nonlinear analysis. While discretizing the model, engineers should be aware of the trade-offs between the accuracy, computational time, and use of the information such as the regions of significant geometric and material nonlinearities. Nonlinear elements should have material behavior to simulate the hysteresis relations under reverse cyclic loading observed in the experiments.

The general issues in modeling of bridge structures include geometry, stiffness, mass distribution, and boundary conditions. In general, abutments, superstructure, bent caps, columns and pier walls, expansion joints, and foundation springs are the elements included in the structural model. The mass distribution in a structural model depends on the number of elements used to represent the bridge components. The model must be able to simulate the vibration modes of all components contributing to the seismic response of the structure.

Superstructure: Superstructure and bent caps are usually modeled using linear elastic three-dimensional beam elements. Detailed models may require nonlinear beam elements.

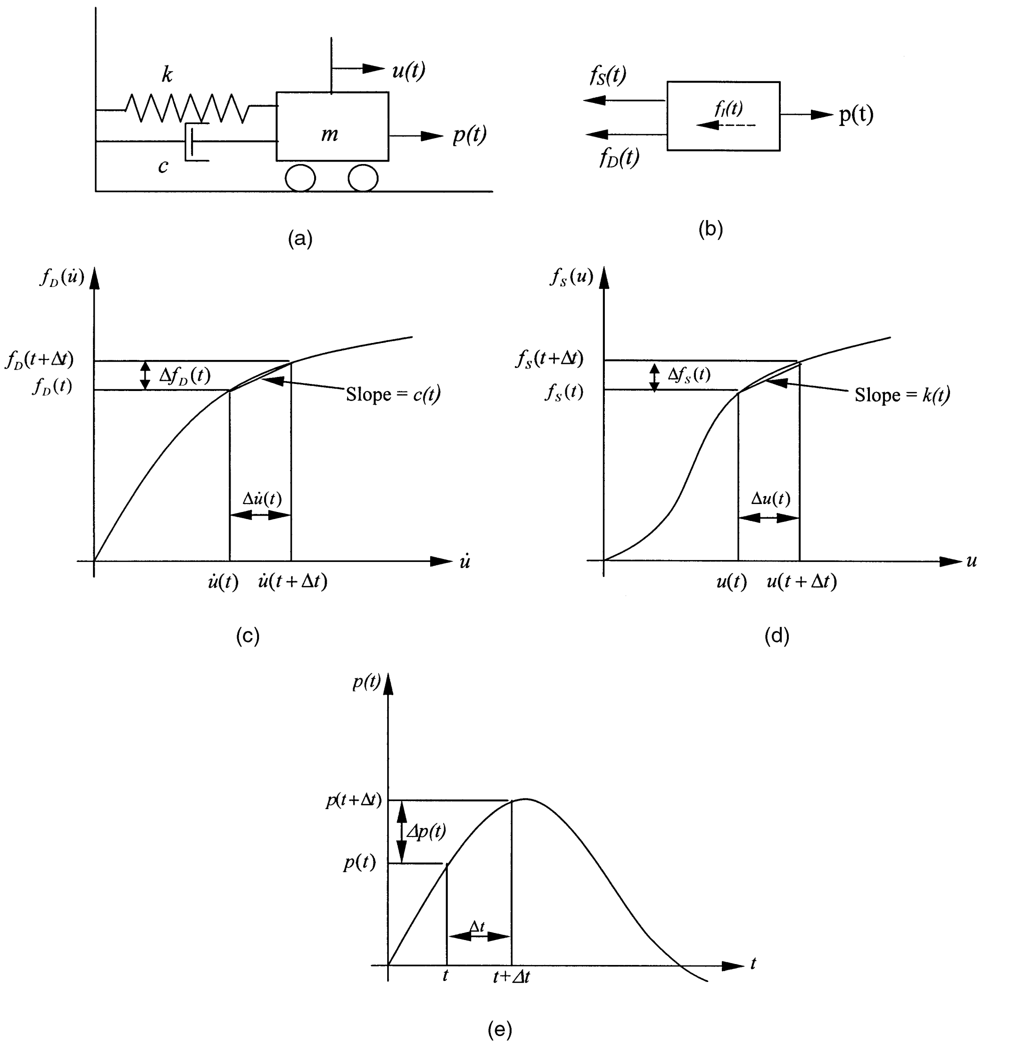

FIGURE 35.20 Definition of a nonlinear dynamic system. (a) Basis SDOF structure; (b) force equilibrium; (c) nonlinear damping; (d) nonlinear stiffness; (e) applied load.

Columns and pier walls: Columns and pier walls are usually modeled using nonlinear beam elements having response properties with a yield surface described by the axial load and biaxial bending. Some characteristics of the column behavior include initial stiffness degradation due to concrete cracking, flexural yielding at the fixed end of the column, strain hardening, pinching at the point of load reversal. Shear actions can be modeled using either linear or nonlinear load deformation relationships for columns. For both columns and pier walls, torsion can be modeled with linear elastic properties. For out-of-plane loading, flexural response of a pier wall is similar to that of columns, whereas for in-plane loading the nonlinear behavior is usually shear action.

Expansion joints: Expansion joints can be modeled using gap elements that simulate the nonlinear behavior of the joint. The variables include initial gap, shear capacity of the joint, and nonlinear load deformation characteristics of the gap.

Foundations and abutments: Foundations are typically modeled using nonlinear spring elements to represent the translational and rotational stiffness of the foundations to represent the expected behavior during a design earthquake. Abutments are modeled using nonlinear spring and gap elements to represent the soil action, stiffness of the pile groups, and gaps at the seat.

35.6 Summary

This chapter has presented the basic principles and methods of dynamic analysis for the seismic design of bridges. Response spectrum analysis — the SDOF or equivalent SDOF-based equivalent static analysis — is efficient, convenient, and most frequently used for ordinary bridges with simple configurations. Elastic dynamic analysis is required for bridges with complex configurations. A multisupport response spectrum analysis recently developed by Kiureghian et al. [7] using a lumped-mass beam element mode may be used in lieu of an elastic time history analysis.

Inelastic response spectrum analysis is a useful concept, but the current approaches apply only to SDOF structures. An actual nonlinear dynamic time history analysis may be necessary for some important and complex bridges, but linearized dynamic analysis (dynamic secant stiffness analysis) and inelastic static analysis (static push-over analysis) (Chapter 36) are the best possible alternatives [8] for the most bridges.

References

1. Clough, R.W. and Penzien, J., *Dynamics of Structures,* 2nd ed., McGraw-Hill, New York, 1993.
2. Caltrans, Guidelines for Generation of Response — Spectrum-Compatible Rock Motion Time History for Application to Caltrans Toll Bridge Seismic Retrofit Projects, the Caltrans Seismic Advisor Board ad hoc Committee on Soil–Foundation-Structure Interaction, California Department of Transportation, Sacramento, 1996.
3. Caltrans, Guidelines for Performing Site Response Analysis to Develop Seismic Ground Motions for Application to Caltrans Toll Bridge Seismic Retrofit Projects, The Caltrans Seismic Advisor Board ad hoc Committee on Soil–Foundation-Structure Interaction, California Department of Transportation, Sacramento, 1996.
4. AASHTO, *LRFD Bridge Design Specifications*, American Association of State Highway and Transportation Officials, Washington, D.C., 1994.
5. AASHTO, *LRFD Bridge Design Specifications*, 1996 Interim Version, American Association of State Highway and Transportation Officials, Washington, D.C., 1996.
6. Wilson, E. L, der Kiureghian, A., and Bayom, E. P., A replacement for SSRS method in seismic analysis, *J. Earthquake Eng. Struct. Dyn.*, 9, 187, 1981.
7. Kiureghian, A. E., Keshishian, P., and Hakobian, A., Multiple Support Response Spectrum Analysis of Bridges Including the Site-Response Effect and the MSRS Code, Report No. UCB/EERC-97/02, University of California, Berkeley, 1997.
8. Powell, G. H., Concepts and Principles for the Application of Nonlinear Structural Analysis in Bridge Design, Report No.UCB/SEMM-97/08, Department of Civil Engineering, University of California, Berkeley, 1997.
9. Chmielewski, T., Kratzig, W. B., Link, M., Meskouris, K., and Wunderlich, W., Phenomena and evaluation of dynamic structural responses, in *Dynamics of Civil Engineering Structures*, W. B. Kratzig and H.-J. Niemann, Eds., A.A. Balkema, Rotterdams, 1996.
10. Chopra, A.K., *Dynamics of Structures*, Prentice-Hall, Englewood Cliffs, NJ, 1995.
11. Lindeburg, M., *Seismic Design of Building Structures: A Professional's Introduction to Earthquake Forces and Design Details*, Professional Publications, Belmont, CA, 1998.
12. Caltrans, Bridge Design Specifications, California Department of Transportation, Sacramento, CA, 1991.
13. ADINA, *User's Guide*, Adina R&D, Inc., Watertown, MA, 1995.
14. GTSTRUDL, *User's Manual*, Georgia Institute of Technology, Atlanta, 1996.
15. SAP2000, *User's Manual*, Computers and Structures Inc., Berkeley, CA, 1998.
16. ANSYS, *User's Manual*, Vols. 1 and 2, Version 4.4, Swanson Analysis Systems, Inc., Houston, TX, 1989.
17. NASTRAN, *User's Manual*, MacNeil Schwindler Corporation, Los Angeles, CA.

18. Idriss, I. M., Sun J. I., and Schnabel, P. B., User's manual for SHAKE91: a computer program for conducting equivalent linear seismic response analyses of horizontally layered soil deposits, *Report of Center for Geotechnical Modeling*, Department of Civil and Environmental Engineering, University of California at Davis, 1991.
19. Abrahamson, N. A., Schneider, J. F., and Stepp, J. C., Empirical spatial coherency functions for application to soil-structure interaction analysis, *Earthquake Spectra*, 7, 1991.
20. Imbsen & Associates, *Seismic Design of Highway Bridges*, Sacramento, CA, 1992.
21. Priestly, M. J. N., Seible, F., and Calvi, G. M., *Seismic Design and Retrofit of Bridges*, John Wiley & Sons, New York, 1996.
22. Bathe, K.-J., *Finite Element Procedures in Engineering Analysis*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1996.
23. ATC 32, Improved Seismic Design Criteria for California Bridges: Provisional Recommendations, Applied Technology Council, 1996.
24. Buchholdt, H. A., *Structural Dynamics for Engineers*, Thomas Telford, London, 1997.
25. Paz, M., *Structural Dynamics — Theory and Computation*, 3rd ed., Van Nostrand Reinhold, New York, 1991.